Problem 1. Assume \( A, B, A_\alpha \subset X \). Prove the following
- If \( A \subset B \) then \( \overline{A} \subset \overline{B} \).
- \( \overline{A \cup B} = \overline{A} \cup \overline{B} \).
- \( \bigcup \overline{A_\alpha} \supset \bigcup \overline{A_\alpha} \), show that equality is not necessary true.

Problem 2. Show that every ordered topology is Hausdorff.

Problem 3. Let \( A \subset X \) and \( B \subset Y \). Show that in the space \( X \times Y \)
\( \overline{A \times B} = \overline{A} \times \overline{B} \)

Problem 4. Find a function \( f : \mathbb{R} \rightarrow \mathbb{R} \), which is continuous at precisely one point.

Problem 5. Let \( Y \) be an ordered set in the ordered topology. Let \( f, g : X \rightarrow Y \) be continuous
- Show that the set \( \{x : f(x) \leq g(x)\} \) is closed in \( X \).
- Let \( h : X \rightarrow Y \) be the function \( h(x) = \min\{f(x), g(y)\} \). Show that \( h(x) \) is continuous.

Problem 6. Let \( f : A \rightarrow B \) and \( g : C \rightarrow D \) be continuous functions. Let us define a map \( f \times g : A \times C \rightarrow B \times D \) by the equation
\( f \times g(a \times c) = f(a) \times g(c) \).
Show that \( f \times g \) is continuous.

Problem 7. Prove that if each space \( X_\alpha \) is a Hausdorff space then \( \prod X_\alpha \) is a Hausdorff space in both the box and product topologies.

Problem 8. Given sequences \( (a_1, a_2, \ldots) \) and \( (b_1, b_2, \ldots) \) of real numbers with \( a_i > 0 \) for all \( i \), define \( h : \mathbb{R}^\omega \rightarrow \mathbb{R}^\omega \) as
\( h((x_1, x_2, \ldots)) = (a_1 x_1 + b_1, a_2 x_2 + b_2, \ldots) \).
Show that if \( \mathbb{R}^\omega \) is is given the product topology then \( h \) is a homeomorphism of \( \mathbb{R}^\omega \) with itself. What happens if \( \mathbb{R}^\omega \) is given the box topology?