Problem 1. Consider the product, uniform and the box topologies on $\mathbb{R}^\omega$. In which topologies are the following functions from $\mathbb{R} \to \mathbb{R}^\omega$ continuous?
- $f(t) = (t, 2t, 3t, \ldots)$.
- $g(t) = (t, t, t, t, \ldots)$.
- $h(t) = (t, \frac{1}{2}t, \frac{1}{3}t, \ldots)$.

Problem 2. In $\mathbb{R}^n$ define $d_1(x, y) = \|x - y\|_1 = \sum_{i=1}^{n} |x_i - y_i|$.
- Show that this is a metric.
- Prove that this induces the usual topology in $\mathbb{R}^n$.
- Consider the $X \subset \mathbb{R}^\omega$, where $X$ is set of all sequences so that $\sum |x_i|$ converges. On $X$ we have three topologies it inherits from the box, uniform and product topologies on $\mathbb{R}^\omega$. We have also a topology given by metric $d_1$. Please compare those 4 topologies!

Problem 3. Show that $\mathbb{R} \times \mathbb{R}$ in dictionary order topology is metrizable.

Problem 4. Prove that if $X$ is a metric space with metric $d$ then $d : X \times X \to \mathbb{R}$ is a continuous function.

Problem 5. Show that if $d$ is a metric on $X$, the
$$d'(x, y) = \frac{d(x, y)}{1 + d(x, y)}$$
is bounded metric that gives the topology of $X$.

Problem 6. Define $f_n : [0, 1] \to \mathbb{R}$ as $f_n(x) = x^n$. Show that the sequence $(f_n(x))$ converges for $x \in [0, 1]$, but does not converge uniformly.