1. Given three vertices of a parallelogram \( z_1, z_2, z_3 \) in \( \mathbb{C} \), find the fourth vertex \( z_4 \), opposite to the vertex \( z_2 \), in terms of the other three vertices.

2. Let \( f : \mathbb{C} \to \mathbb{C} \) be an entire function. Calculate each of the following integrals, in terms of the function \( f \) and its derivatives. (Here, \( C_s(b) = \{ z \in \mathbb{C} : |z-b| = s \}. \)

   \[(a). \quad \frac{1}{2\pi i} \int_{C_r(a)} \frac{f^2(z)}{(z-a)^2} \, dz,\]
   \[(b). \quad \left[ \frac{1}{2\pi i} \int_{C_r(a)} \frac{f(z)}{z-a} \, dz \right]^2,\]
   \[(c). \quad \frac{1}{2\pi i} \int_{C_r(a)} \frac{f(z)}{(z-a)^2} \, dz,\]
   \[(d). \quad \frac{1}{2\pi i} \int_{C_r(a)} \frac{f^2(z)}{z-a} \, dz.\]

3. Let \( f \) be a meromorphic function on \( \mathbb{C} \), having poles at the three points \( z = 1 + 3i, 3 - 4i, \) and \( 5 \), as well as one removable singularity at \( z = 3 \). In each case below, either provide the requested quantity (with explanation) or explain why not enough information has been provided to find this quantity.

   \[(a). \limsup_{n \to \infty} (\frac{|f^{(n)}(3+i)|}{n})^{1/n}.\]
   \[(b). \lim_{z \to 5} |f(z)|.\]
   \[(c). \lim_{z \to 1+3i} (z - 1 - 3i)f(z).\]
   \[(d). \lim_{z \to \infty} |f(z)|.\]

4. Let \( f : D \to D \) be an analytic function, where \( D \) is the open unit disc in \( \mathbb{C} \). Suppose that \( f(1/4) = -2/3 \). Is it possible for \( f(1/3) = 2/3 \)? Explain your reasoning.

5. Let \( f : D \to D \) be an analytic function, where \( D \) is the open unit disc in \( \mathbb{C} \). Suppose that there is a positive number \( \delta > 0 \) such that for every \( \theta, \ |\theta| < \delta, \lim_{z \to e^{i\theta}} f(z) = 0 \). Prove that \( f \equiv 0 \) on \( D \).
6. (a). State the Cauchy-Riemann equations and prove that if \( f \) is differentiable at a point \( b \in \mathbb{C} \), then the Cauchy-Riemann equations hold at \( b \).

(b). Let \( f : U \to \mathbb{C} \) be an analytic function on the domain \( U \) such that \( \text{Re} \, f(z) = (\text{Im} \, f(z))^2 \) for all \( z \in U \). Prove that \( f \) is a constant function.

7. Find all real numbers \( b \) so that the following integral exists, and for these \( b \) evaluate this integral:

\[
\int_{-\infty}^{\infty} \frac{1}{x^2 + bx + 1} \, dx.
\]

8. Let \( P(z) = 2z^4 + 5z^2 \) and \( Q(z) = z^4 + 10z^2 + 1 \). Prove that \( P \) and \( Q \) have the same number of zeros inside the open unit disc as well as the same number of zeros outside the unit disc but inside the disc of radius 4 centered at 0.

9. Find the linear fractional transformation which transforms the points \( -1, 0, 1 \) respectively into the points \( 1, i, -1 \), and explain what the upper half-plane becomes in this mapping.

10. Find the domains of convergence of the given series.

(a).
\[
\sum_{n=0}^{\infty} \left( z^n + \frac{1}{2^n z^n} \right),
\]

(b).
\[
\sum_{n=0}^{\infty} \frac{(-1)^n}{z + n}.
\]