Directions

1. Answer questions completely: a fully-done problem is far more revealing of your understanding than two half-done attempts.

2. State your reasons for your claims. You may cite and use standard results. However, you have to demonstrate that you know why things are so; you cannot assume that we know that you know. That’s the purpose of this examination! An exercise that states a standard result of the theory requires that you supply a proof.
1. Let $F$ be an increasing function on the real line. Then the set of discontinuities of $F$ is countable.

2. A point $x$ is said to belong to the support of the distribution function if for every $a > 0$ we have $F(x + a) - F(x - a) > 0$. Prove that the support of any distribution function is a closed set.

3. Count the number of subsets of an $n$-point set.

4. If $F$ is a Borel field generated by a countable collection of pairwise disjoint sets, then each member of $F$ is just the union of a subcollection of these sets.
5. If two random variables are equal almost everywhere, then they have the same probability measure.

6. The random variable $X$ is independent of itself if and only if it is constant with probability one.