Exercise 1.4 Consider the model with \( K = 3, \ N = 2, \ r = 0 \), and the following security prices:

<table>
<thead>
<tr>
<th>( n )</th>
<th>( S_n(0) )</th>
<th>( S_n(1)(\omega_1) )</th>
<th>( S_n(1)(\omega_2) )</th>
<th>( S_n(1)(\omega_3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>8</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>10</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

Show that there exist dominant trading strategies and that the law of one price holds.

Exercise 1.5 Show for example 1.3 that there are no dominant trading strategies but there exists an arbitrage opportunity.

\[ \text{Example 1.3 (Revised)} \]

\[ \begin{array}{cccc}
K = 3, \ n = 4/
\end{array} \]

It follows that the discounted price process is given by

<table>
<thead>
<tr>
<th>( n )</th>
<th>( S_n(0) )</th>
<th>( S_n(1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_1 )</td>
<td>( \omega_2 )</td>
<td>( \omega_3 )</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>60/9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>40/3</td>
</tr>
</tbody>
</table>

The other quantities of interest are left to the reader.

Exercise 1.8 Determine either all the risk neutral probability measures or all the arbitrage opportunities in the case of example 1.4.

Exercise 1.9 Suppose \( K = 2, \ N = 1, \) and the interest rate is a scalar parameter \( r \geq 0 \). Also, suppose \( S_0 = 1, \ S_1(\omega_1) = u \) ("up"), and \( S_1(\omega_2) = d \) ("down"), where the parameters \( u \) and \( d \) satisfy \( u > d > 0 \). For what values of \( r, u, \) and \( d \) does there exist a risk neutral probability measure? Say what this measure is. For the complementary values of these parameters, say what all the arbitrage opportunities are.

Exercise 1.10 Let \( A \) denote the \((K + 1) \times (K + 2N)\) matrix

\[
\begin{bmatrix}
0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 \\
\Delta S_1^T(\omega_1) & -\Delta S_1^T(\omega_1) & \Delta S_1^T(\omega_1) & \cdots & -\Delta S_N^T(\omega_1) & -1 & 0 & \cdots & 0 \\
\Delta S_1^T(\omega_2) & -\Delta S_1^T(\omega_2) & \Delta S_2^T(\omega_2) & \cdots & -\Delta S_N^T(\omega_2) & 0 & -1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\Delta S_1^T(\omega_K) & -\Delta S_1^T(\omega_K) & \Delta S_K^T(\omega_K) & \cdots & -\Delta S_N^T(\omega_K) & 0 & 0 & \cdots & -1
\end{bmatrix}
\]

and let \( b \) denote the \((K + 1)\)-component column vector \((1, 0, \ldots, 0)'\). Show that

\[ Ax = b, \quad x \geq 0, \quad x \in \mathbb{R}^{K+2N} \]

has a solution if and only if there exists an arbitrage opportunity.

Example 1.1 (continued) With \( r = 1/9 \) and \( e = \frac{1}{5} \), the time \( t = 6 \) value of the call option is:

\[ \text{European call option} \]