Section 1.1 Some basic math models and direction fields 15-20, 25

15. (j) $y' = 2 - y$, since $y = 2$ is equilibrium, and when $y > 2$, $y' < 0$, and when $y < 2$, $y' > 0$,
16. (c) $y' = y - 2$, since $y = 2$ is equilibrium, and when $y > 2$, $y' > 0$, and when $y < 2$, $y' < 0$.
17. (g) $y = -y - 2$, since $y = -2$ is equilibrium, and when $y > -2$, $y' < 0$ and when $y < -2$, $y' > 0$.
18. (b) $y' = y + 2$, since $y = -2$ is equilibrium, and when $y > -2$, $y' < 0$ and when $y' < -2$, $y' < 0$.
19. (h) $y' = y(3 - y)$, since $y = 0$ and $y = 3$ are two equilibriums, and when $y > 3$, $y' < 0$ and when $0 < y < 3$, $y' > 0$, when $y' < 0$, $y' < 0$.
20. (e) $y' = y(y - 3)$, since $y = 0$ and $y = 3$ are two equilibriums, and when $y > 3$, $y' > 0$, $0 < y < 3$, $y' < 0$ and $y < 0$, $y' > 0$.

25. (a) Following the discussion in the text, the differential equation is

$$\frac{dv}{dt} = mg - \gamma v^2$$

or equivalently,

$$\frac{dv}{dt} = g - \frac{\gamma}{m} v^2.$$  

(b) After a long time, $\frac{dv}{dt} \approx 0$. Hence the object attains a terminal velocity given by

$$v_\infty = \sqrt{\frac{mg}{\gamma}}.$$

(c) Using the relation $\gamma v_\infty^2 = mg$, the required drag coefficient is $\gamma = 2/49$ kg/s.

(d)
1. The differential equation is second order, since the highest derivative in the equation is of order two. The equation is linear, since the left hand side is a linear function of $y$ and its derivatives.

2. The differential equation is second order, since the highest derivative of the function $y$ is of order two. The equation is nonlinear due to $y^2$ term, as well as due to the $y^3$ term multiplying the $y''$ term.

3. The differential equation is fourth order, since the highest derivative of the function $y$ is of order four. The equation is also linear, since the terms containing the dependent variable is linear in $y$ and its derivatives.

4. The differential equation is first order, since the only derivative is of order one. The dependent variable is squared, hence the equation is nonlinear.

5. The differential equation is second order. Furthermore, the equation is nonlinear, since the dependent variable $y$ is an argument of the sine function, which is not a linear function.

6. This is a third order differential equation, since the highest derivative is $y'''$, and it is linear since $y$ and all its derivatives appear to the first power only. The term $t^3$ and $\cos t$ do not affect the linearity of the differential equation.

7. $y_1(t) = e^t \Rightarrow y_1'(t) = y_1''(t) = e^t$. Hence $y_1'' - y_1 = 0$. Also, $y_2(t) = \cosh t \Rightarrow y_2'(t) = \sinh t$ and $y_2''(t) = \cosh t$. Thus $y_2'' - y_2 = 0$. 
