GENERALIZATIONS OF VERTEX ALGEBRAS

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A vertex algebra is, roughly speaking, a linear space $A$ that has infinitely many bilinear operations $(a, b) \rightarrow a(n)b$, indexed by integer number $n$, satisfying certain identities. It is natural to consider the generating function for these products:

$$Y(a, z) = ... + a(-2)z + a(-1) + a(0)z^{-1} + a(1)z^{-2} + ...$$

where $a(n) : A \rightarrow A$ is the operator of $n$-th left multiplication by $a$. One of the main properties of vertex algebras is that for any $a_1, ..., a_k, b \in A$ and a functional $f : A \rightarrow C$, the series

$$f(Y(a_1, z_1)Y(a_2, z_2)...Y(a_l, z_k)b), \quad a_i, b \in A$$

converge in some region of $C^k$ to a rational function in $z_1, ..., z_k$. We consider a more general class of algebras, such that the above series converge to an analytic function in $k$ variables, which is not necessarily rational. We outline how these structures appear in representation theory and geometry, and give a method of constructing such algebras starting from certain spaces of complex analytic functions.

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