

# Who Should be the Next Math Club President?

## *The Mathematics of Democratic Voting*

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# How do you become Math Club King, I mean, President?



**King Arthur:** I am your king.

**Peasant Woman:** Well, I didn't vote for you.

**King Arthur:** You don't vote for kings.

**Peasant Woman:** Well, how'd you become king, then?



## Math Club Election: Plurality

- ▶ 4 Candidates: Ann, Bob, Cate, Don
- ▶ 29 club members vote for their top choice
- ▶ Results:

Candidate	Ann	Bob	Cate	Don
# votes	11	3	8	7
%	37.9%	10.3%	27.6%	24.1%

- ▶ Ann is declared the winner using the *Plurality Method*
- ▶ Note that no candidate earns a *Majority* of votes ( $> 50\%$ )
- ▶ Does this really reflect the will of the people?



## Math Club Election: Antiplurality

- ▶ Bob wonders, “How can this be? Everyone I know *hates* Ann!”
- ▶ He suggests the club members vote *against* their bottom choice
- ▶ Results:

Candidate	Ann	Bob	Cate	Don
# votes against	18	0	0	11
% against	62.1%	0%	0%	37.9%

- ▶ Bob and Cate are tied for president using the *Antiplurality Method*
- ▶ Cate is not happy with the tie



# Math Club Election: Plurality with Elimination, Version I

- ▶ Cate suggests eliminating Ann and then revoting, removing the candidate with most last-place votes, etc., until one candidate remains
- ▶ Easiest to cast ballots with full rankings one time
- ▶ *Preference Schedule:*

# voters	11	7	7	3	1
1st place	Ann	Cate	Don	Bob	Cate
2nd place	Bob	Bob	Cate	Don	Don
3rd place	Cate	Don	Bob	Cate	Bob
4th place	Don	Ann	Ann	Ann	Ann

- ▶ Ann has the most last-place votes, so she is eliminated



# Math Club Election: Plurality with Elimination, Version I

- ▶ Ann is removed from the ballots and they are recounted
- ▶ Results:

# voters	11	7	7	3	1
1st place	Bob	Cate	Don	Bob	Cate
2nd place	Cate	Bob	Cate	Don	Don
3rd place	Don	Don	Bob	Cate	Bob

- ▶ Now Don has the most last-place votes (18), so he is eliminated



# Math Club Election: Plurality with Elimination, Version I

- ▶ Don is removed from the ballots and they are recounted
- ▶ Results:

<b># voters</b>	<b>11</b>	<b>7</b>	<b>7</b>	<b>3</b>	<b>1</b>
<b>1st place</b>	Bob	Cate	Cate	Bob	Cate
<b>2nd place</b>	Cate	Bob	Bob	Cate	Bob

- ▶ Now Bob has the most last-place votes (15), so he is eliminated and Cate is the winner!



## Math Club Election: Plurality with Elimination, Version II

- ▶ Not so fast, says Don
- ▶ Instead of eliminating the candidate with the *most last-place* votes, we should eliminate the one with the *fewest first-place* votes
- ▶ Here that would be Bob with only 3 first-place votes

# voters	11	7	7	3	1
1st place	Ann	Cate	Don	Bob	Cate
2nd place	Bob	Bob	Cate	Don	Don
3rd place	Cate	Don	Bob	Cate	Bob
4th place	Don	Ann	Ann	Ann	Ann





## Math Club Election: Plurality with Elimination, Version II

- ▶ Remove Bob from the ballots and recount
- ▶ Results:

# voters	11	7	7	3	1
1st place	Ann	Cate	Don	Don	Cate
2nd place	Cate	Don	Cate	Cate	Don
3rd place	Don	Ann	Ann	Ann	Ann

- ▶ Now Cate has fewest first-place votes (8)



## Math Club Election: Plurality with Elimination, Version II

- ▶ Remove Cate from the ballots and recount
- ▶ Results:

<b># voters</b>	<b>11</b>	<b>7</b>	<b>7</b>	<b>3</b>	<b>1</b>
<b>1st place</b>	Ann	Don	Don	Don	Don
<b>2nd place</b>	Don	Ann	Ann	Ann	Ann

- ▶ Now Ann has fewest first-place votes (11), so she is eliminated and Don is the winner!
- ▶ This method is sometimes called *Instant Run-Off Voting (IRV)*



## Math Club Election: Borda Count

- ▶ Bob suggests using a point system.

	pts/vote	11	7	7	3	1
<b>1st place</b>	<b>3</b>	Ann	Cate	Don	Bob	Cate
<b>2nd place</b>	<b>2</b>	Bob	Bob	Cate	Don	Don
<b>3rd place</b>	<b>1</b>	Cate	Don	Bob	Cate	Bob
<b>4th place</b>	<b>0</b>	Don	Ann	Ann	Ann	Ann

- ▶ **Ann:**  $11 \times 3 = 33$
- ▶ **Bob:**  $(3 \times 3) + (18 \times 2) + (8 \times 1) = 53$
- ▶ **Cate:**  $(8 \times 3) + (7 \times 2) + (14 \times 1) = 52$
- ▶ **Don:**  $(7 \times 3) + (4 \times 2) + (7 \times 1) = 36$
- ▶ So Bob is the winner!



# Math Club Election: Pairwise Comparisons

- ▶ Cate notes that she would beat each of the other candidates in a head-to-head contest

# voters	11	7	7	3	1
1st place	Ann	Cate	Don	Bob	Cate
2nd place	Bob	Bob	Cate	Don	Don
3rd place	Cate	Don	Bob	Cate	Bob
4th place	Don	Ann	Ann	Ann	Ann

- ▶ Cate beats Ann 18 to 11
- ▶ Cate beats Bob 15 to 14
- ▶ Cate beats Don 19 to 10
- ▶ Cate is therefore a *Condorcet Winner*



“It’s not the voting that’s democracy, it’s the counting.”

– Dotty, in Tom Stoppard’s play *Jumpers*

The crux of the matter:

How do we aggregate individual voters’ preferences to produce a societal preference in the fairest way possible?

What is “fair”?



## Fairness Criteria: The Majority Criterion

### Definition (The Majority Criterion.)

If a candidate receives a majority ( $> 50\%$ ) of the first-place votes, that candidate should be a winner of the election.

- ▶ Violated by Borda Count

	pts/vote	3	2
<b>1st place</b>	<b>2</b>	A	B
<b>2nd place</b>	<b>1</b>	B	C
<b>3rd place</b>	<b>0</b>	C	A

- ▶ A:  $(3 \times 2) = 6$
- ▶ B:  $(2 \times 2) + (3 \times 1) = 7$
- ▶ C:  $(2 \times 1) = 2$
- ▶ A has a majority, but B wins under Borda Count



### Definition (The Condorcet Criterion.)

If a candidate beats each other candidate in a pairwise comparison, that candidate should be a winner of the election.

- ▶ Violated by Plurality, Instant Run-Off Voting, and Borda Count
- ▶ Cate was Condorcet Candidate in Math Club Election, but lost using Plurality, Instant Run-off Voting, and Borda Count



## Fairness Criteria: The Monotonicity Criterion

### Definition (The Monotonicity Criterion.)

If candidate  $X$  is a winner, then  $X$  should remain a winner if a voter moves  $X$  (and only  $X$ ) up on his/her ballot.

- ▶ Violated by Instant Run-Off Voting

	<b>7</b>	<b>8</b>	<b>10</b>	<b>2</b>
<b>1st place</b>	A	B	C	A
<b>2nd place</b>	B	C	A	C
<b>3rd place</b>	C	A	B	B

- ▶ C wins: B is eliminated in the first round and B's 8 votes get transferred to C (who now has 18/27)
- ▶ Now suppose the last two voters want to vote for the winner (C), so they change their votes, moving C up





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	<b>7</b>	<b>8</b>	<b>10</b>	<b>2</b>
<b>1st place</b>	A	B	C	C
<b>2nd place</b>	B	C	A	A
<b>3rd place</b>	C	A	B	B

- ▶ B wins: A is eliminated in the first round and A's 7 votes get transferred to B, who beats C 15 to 12.



## Definition (Independence of Irrelevant Alternatives Criterion.)

If candidate  $X$  is a winner, then  $X$  should remain a winner if any of the irrelevant (losing) candidates drops out of the race.

- ▶ All of the voting methods we've seen violate the Independence of Irrelevant Alternatives Criterion!



# Transitivity (or lack thereof)

## Definition (Transitivity)

If I prefer  $P$  to  $R$  and  $R$  to  $S$ , it is reasonable to assume I prefer  $P$  to  $S$ . (Write  $P > R > S$ )

- ▶ Suppose there are two other voters with transitive preferences  $R > S > P$  and  $S > P > R$
- ▶ Preference schedule:

# voters	1	1	1
1st place	Paper	Rock	Scissors
2nd place	Rock	Scissors	Paper
3rd place	Scissors	Paper	Rock

- ▶ This is a tie, but it's worse than that— it's a *Cycle*.
- ▶ Pairwise comparison rankings are *Intransitive*
  - ▶ ( $P > R$ ): Paper beats Rock 2 to 1
  - ▶ ( $R > S$ ): Rock beats Scissors 2 to 1
  - ▶ ( $S > R$ ): Scissors beats Paper 2 to 1



# Arrow's Impossibility Theorem

Theorem (Arrow's Impossibility Theorem)

*Any transitive voting method that satisfies all of these fairness criteria is a dictatorship.*



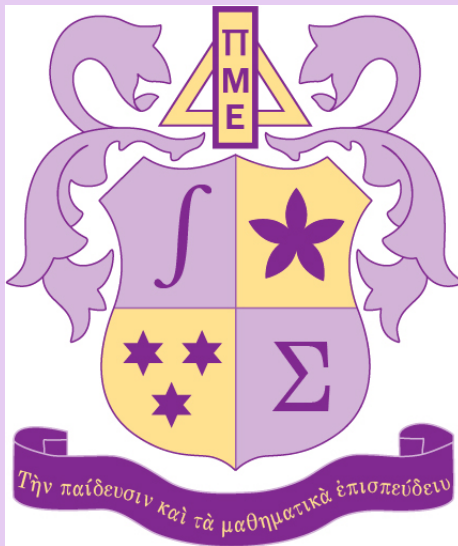
# Conclusions



**Count de Money:** Your majesty, it is said that the people are revolting.

**King Louis XVI:** You said it. They stink on ice!





## The Society's Goals:

- ▶ To elect members on an honorary basis according to their proficiency in mathematics
- ▶ To promote activities that enhance the mathematical and scholarly development of its members



# The History of Pi Mu Epsilon

- ▶ Founded at Syracuse University on December 8th, 1913
- ▶ Named using Greek letters stemming from the Greek words for scholarship (Pi), mathematics (Mu), and promotion (Epsilon)
- ▶ Incorporated on May 25th, 1914. Re-incorporated in 1988.
- ▶ A national society comprised of local chapters at colleges and universities.
- ▶ Currently there are 389 chapters in 48 states and the District of Columbia. (Missing: Wyoming and Hawaii)
- ▶ Each chapter is designated by its own Greek Letter and a chapter number.
- ▶ The Pennsylvania Upsilon Chapter (chapter 287) was chartered at Duquesne University in 1999





# The Society Council

- ▶ Angela Spalsbury, Ohio Xi at Youngstown State University (President)
- ▶ Paul Fishback, Michigan Iota at Grand Valley State University (President-Elect)
- ▶ Eve Torrence, Virginia Iota at Randolph-Macon College (Past-President)
- ▶ Stephanie Edwards, Michigan Delta at Hope College (Secretary-Treasurer)
- ▶ Brigitte Servatius, Massachusetts Alpha at Worcester Polytechnic Institute (Journal Editor)
- ▶ Councilors
  - ▶ Chad Awtrey, North Carolina Nu at Elon University
  - ▶ Jennifer Beineke, Massachusetts Kappa at Western New England University
  - ▶ Darci Kracht, Ohio Epsilon at Kent State University
  - ▶ Ben Ntatin, Tennessee Epsilon at Austin Peay University



# Activities of the National Organization

Financial support for various organizations:

- ▶ American Mathematics Competitions
- ▶ American Regional Mathematics League
- ▶ Mathematical Association of America (MAA) National Meeting Poster Session



# Activities of the National Organization

## Chapter Grants:

- ▶ Matching Prize Grants (\$100)
- ▶ Matching Conference Grants (\$300)
- ▶ Richard A. Good Lectureship Grants (\$500)



# Activities of the National Organization

## *The Pi Mu Epsilon Journal*



- ▶ Published in the fall and spring of each year
- ▶ Cash prizes for student-authored articles

“This award had a MAJOR impact on my vision for a research career.” Robert Devaney, Boston University, MAA Past-President



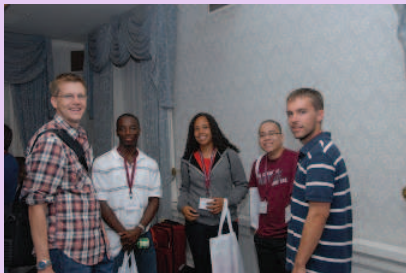
# The 2016 National Pi Mu Epsilon Conference



in conjunction with MAA MathFest 2016  
August 3–6  
Columbus, OH



# PME-MAA Opening Student Reception



# Student Presentations at the National PME Meeting

Fifteen-minute talks may be expository on material most undergraduates have not seen in their classrooms or on new research accomplished while an undergraduate.



# Student Presentations at the National PME Meeting

Sample titles from previous conferences:

- ▶ Computational Models of Congressional Redistricting
- ▶ Exploring Leibniz's Infinitesimals
- ▶ Integer Compositions Applied to the Probability Analysis of Blackjack and Infinite Deck Assumption
- ▶ A Quantitative Analysis of SIR-type Malaria Models
- ▶ Mathematical Manipulatives from 3D Printing
- ▶ Using Independent Bernoulli Random Variables to Model Gender Hiring Practices





# Activities Sponsored by the Mathematical Association of America

- ▶ Math Jeopardy
- ▶ *Zombies and Calculus: A Survival Guide*, Colin Adams, Williams College
- ▶ *Games Mathematicians Play*, Christopher Swanson, Ashland University
- ▶ Panel Session: *Non-Academic Mathematical Career Paths for Undergraduates*
- ▶ Estimation!
- ▶ Student Poster Sessions and Other Undergraduate Activities



# Pi Mu Epsilon Banquet and Awards Ceremony



## Awards for Student Talks at the National PME Meeting



Talks are judged, and cash prizes (\$150) are awarded by several professional organizations:

- ▶ The American Mathematics Society
- ▶ The MAA Special Interest Groups on Mathematical Biology and Environmental Mathematics
- ▶ The American Statistical Association
- ▶ The Society for Industrial and Applied Mathematics
- ▶ Budapest Semesters in Mathematics





Professor Robin Wilson, Open University  
*Combinatorics—The Mathematics That Counts*



## Travel Funding for the National PME Meeting

- ▶ PME provides transportation support for up to 5 student speakers from each Chapter: up to \$600 per student with a \$1200 per Chapter maximum.
- ▶ An NSA grant provides a stipend to help defray lodging and food expenses. (in 2014: \$380 each)
- ▶ **Almost all PME student speakers receive travel and sustenance grants.**
- ▶ For further details, see [www.pme-math.org/apply-for-funding](http://www.pme-math.org/apply-for-funding).



# 2014 Student Speakers

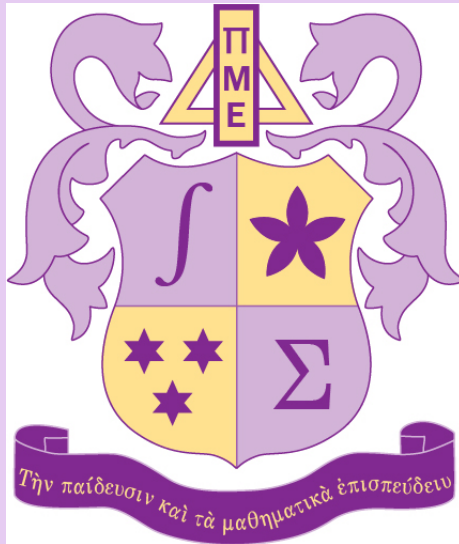


## Student Survey Comments

- ▶ “MathFest is an excellent opportunity to expand your mathematical knowledge, meet distinguished mathematicians, and learn about careers in your field. It was a wonderful and fun experience and you should definitely participate.”
- ▶ “I would tell students that they should participate and give a talk. It has been a memorable and great experience that will help me in several ways in the future.”



# The Meaning of the Shield:





## The Pi Mu Epsilon Pledge:

I solemnly promise  
that I will exert my best efforts  
to promote true scholarship,  
particularly in mathematics,  
and that I will support the objectives  
of the Pi Mu Epsilon Honor Society.

