Instructor Notes for Modules 1 & 2

M1 I1

This module is a review of basic skills.

Learning outcomes. Students will:

Use the order of operations conventions correctly in calculations;
Evaluate an absolute value expression by hand;
Multiply a monomial by a polynomial;
Divide a polynomial by a monomial;
Simplify radicals to obtain an equivalent expression.

Suggested order of lesson:

1. Have students work on #1, #2, #3, #5, #8, and #14 in the workbook.
2. Then review with them: order of operations, distance on the number line and absolute value, simplifying expressions (like #5 and #6), and radicals.

Spend the most time on simplifying radicals, since your students are likely to be weakest on this skill.

HW due next class: HW1 and PC1 on IMathAS

M1 I2 & I3. Evaluating Formulas; Relating Quantities and Solving Equations

These investigations ask students to evaluate given formulas and write formulas for a scenario in context. Students will also review solving simple equations.

Learning outcomes. Students will:

Write and evaluate formulas for perimeter and areas of rectangles, and diameter and area of a circle.
Make a sketch and write a formula for a given novel real-world scenario, interpret each part of the formula, then evaluate it for different values of the input variable.
Begin to develop a repertoire of problem-solving strategies, including making a sketch, relating to a simpler problem (perhaps numeric), and making a table of values.
Begin to realize that doing mathematics involves thinking, trying multiple approaches, initial confusion, and often incomplete or incorrect first attempts.
Begin to realize that mathematics can and should make sense to them and helps understand their world.

Suggested order of lesson:

1. Ask students to work in small groups on #1 and #2 on pp. 7-8 and #1 and #3 on pp. 11-13. (Problem #2 is a good review, but you probably won't have time for it.) Review and discuss.
2. Then work together as a class to solve the equations in #3, #4 and #6.

HW due next class: HW1B and PC2 on IMathAS

M2 I1 & M2 I2. Quantities, Co-variation in Quantities & Changes in Quantities

Pre-class assignment: The PC1 (Pre-class assignment 1) on IMathAS consists of the following Investigations in the workbook: M2_I1: #1 parts a) – e) and parts i) and j). PC 2 has the following: p. 29 #6, #8a, 8b, 8c.
You might take student answers to the pre-class assignment and use them as a springboard for class discussion.

Learning outcomes. Students will:

Give a definition of quantity and a definition of variable;
Given a variable in context, determine whether it is varying or constant;
Represent constant and varying quantities with expressions involving variables and numbers;
Read and interpret a point on a graph representing a real-world scenario.
Understand the difference between a quantity and *change* in a quantity;
Understand and correctly use notation symbolizing the above (i.e. x and Δx);
Correctly calculate the change in a given quantity given two values of that quantity;

Suggested order of lesson:

1. Review briefly student answers to PC1.
2. Ask students work on #2 and #3 in small groups. Discuss
3. Do together #9, p. 23
4. Students work together: #1 on p. 25. Then discuss it together
5. Instructor discuss #6 (and #7 and/or #10 time permitting) with students.

HW due next class: **Paper and Pencil, to hand in:** #7 on p. 21, #10 and #11 on p. 24; **IMathAS:** HW2 and PC3

M2_I3 Constant rate of change

The Pre-class assignment for this section (PC2) on IMathAS consists of the following Investigations in the workbook:
You might take student answers to the pre-class assignment and use them as a springboard for class discussion.

Learning outcomes. Students will:

Recognize a constant rate of change in a real world scenario and/or from a table;
Determine the constant rate of change in a real world scenario and/or from a table;
Write a formula that expresses the *change* in an output with respect to the *change* in the input $(\Delta y = m\Delta x)$.

These materials don't use the term, *slope*, but rather use the terms *constant rate of change.* Since students are familiar with the notion of slope, I would definitely introduce that term and use the materials in this section to deepen their understanding of it.

Expect students to have some difficulty with the final outcome listed above. You need to work with them to understand the difference between $\Delta y = m\Delta x$ and $y = mx$ $(\text{or } y = mx + b)$. Problem #4 and #5 on PC3 deal with these formulas. Be sure to develop these ideas in context first. Also, if students understand that Δx and Δy mean change in x and change in y, then you might develop the formula for a linear function as follows:

$$\Delta y = m\Delta x$$

but since Δ means change, we have

$$y_2 - y_1 = m(x_2 - x_1)$$

Then simplify to obtain the familiar $y = mx + b$ form.

Suggested order of lesson:

1. Ask students to work on #8, p. 29 in small groups, then discuss. Be sure to generate the formula $\Delta y = 3\Delta x$ and interpret it like this: *The **change** in the cost of chicken is always three times as large as the **change** in the number of pounds of chicken*. This statement is not necessarily intuitive to students. You might do several numeric examples. You also might include one that includes a fixed cost. For example, if there is a parking

fee of $1.00, then it may be more clear that the formula representing the change in the total cost $(\Delta y = 3\Delta x)$ is not the same as the formula representing the total cost $(y = 3x + 1)$. There is a worksheet on our Blackboard course that reinforces this skill.

2. Discuss together #9 on p. 30
3. Ask students work on #2 and #3 on p. 34-35 in small groups, then discuss.
4. Work through #4 – 6 and discuss

HW due next class: **IMathAS**: HW3 and PC4

M2 I4 Constant Rate of Change & Linearity

We have two days for this section.

The Pre-class assignment for this section (PC3) on IMathAS consists of the following Investigations in the workbook: p. 34-35: #3 and #4.

This section continues with the idea of linear function. The desired outcomes are as follows:

Learning outcomes. Students will :

Write a formula that expresses the *change* in an output with respect to the *change* in the input;
Write a linear function that expresses the output value with respect to the input value;
Students understand the difference between a linear function and a proportion and can recognize from a real world scenario, a table, and a graph when the relationship is a proportion.

DAY 1

Suggested order of lesson:

1. Ask students to work through #7 on p. 36. Discuss. Part f) in this problem has them write a linear function.
2. Work through #5 on p. 42 with students.
3. Ask students to work on #6 on p. 42.
4. Work through #7 on p. 43 with students.

HW due next class: **paper and pencil to be handed in:** #8 on p. 44; **IMathAS**: HW4 and PC5

DAY 2

Suggested order of lesson:

1. Review work from yesterday.
2. Work through one example from the worksheet on Blackboard
3. Ask students to complete the worksheet.
4. Emphasize the difference between linearity and proportions. You might spend time reviewing hw problems concerning this topic.

HW due next class: Complete the worksheet if they didn't finish in class; IMathAS: HW4B and PC5

M2 I5 Exploring Average Speed

The Pre-class assignment for this section (PC5) on IMathAS consists of the following Investigations in the workbook: p. 45: #1c, 1d, 1e, 1f.

This section introduces the notion of average rate of change in the context of driving a car.

Learning outcomes. Students will :

Describe average speed (average rate of change) is the constant speed (rate of change) needed to travel the same distance (or whatever unit) over the same period of time;
Write the above definition;
Calculate and interpret the average rate of change of a function in context if given the function formula and values of the input;
Illustrate a constant rate of change on the graph of a non-linear function;
Explain such a graph in a real-world context.

Suggested order of lesson:

1. Ask students to work on #1 on p. 45 then discuss
2. Work through together with students: #2, #4, and #6 on pp. 46 – 48.

Be sure to call your students' attention to the boxed definition on the bottom of page 46.

HW due next class: **IMathAS**: HW5 and PC6

M2 I6 The Distance Formula and the Equation of a Circle

We have two days for this section.

The Pre-class assignment for this section (PC6) on IMathAS consists of the following Investigations in the workbook: p. 51: #1a, 1d, 1e.

This section develops the formula for finding the distance between any two points in the plane and then expands that formula to write the general formula for a circle.

Learning outcomes. Students will be able to

- Explain in their own words the derivation the distance formula if given two points in the plane;
- Apply the distance formula to find the distance between any two points in a table, on a graph, or in context;
- Explain how the distance formula can be used to develop a formula for a circle;
- Apply the formula for circle to answer questions in context;
- Write an equation for a circle given center and radius or given the center and a point on the circle.

DAY 1

I have an application problem I've used for years to develop the distance formula. It has to do with the distance a ball was thrown on a football field, given the location of the quarterback and receiver. I've posted this problem on Blackboard. Alternately, you might use #1 on p. 49. I would simply ask students to find the distance from the house to the cell tower, though, and not drag it out as it is in the workbook. Guide students carefully through the derivation of the general formula using the Pythagorean Theorem. They need some practice here. Problems #86 – 88 on p. 77 are good for that.

Suggested order of lesson:

1. Ask students to work through the football problem OR #1 on p. 49 in small group.
2. Guide students carefully through the derivation of the general formula using the Pythagorean Theorem.
3. Student practice: #86 – 88 on p. 77.

HW due next class: **IMathAS**: HW6

DAY 2

1. Work through #2 on p. 50 with students, then do a few traditional type circle problems together.
2. Skill and drill with circles: #89-93 on p. 77

Be sure to work through a couple problems like #91-93 on p. 77 since students are likely to see a similar problem on quizzes or exams.

HW due next class: **IMathAS**: HW6B and PC7

M2 I7 Absolute Value

We have two days for this section.

The Pre-class assignment for this section (PC6) on IMathAS consists of the following Investigations in the workbook: p. 51: #2a, #2b, #4a, #4b, #10iia, #10iib. You might ask students to discuss their answers to PC 7 in small groups, then ask them to formalize the discussion and develop the formal definition of absolute value. You will probably need to do a few examples of solving the equations and inequalities together.
This section develops the notion of absolute value as distance on the number line.

Learning outcomes. Students will be able to

- Describe absolute value as giving distance on the number line;
- List several values that meet a constraint such as, "find an *x* value *within y* units of a given number;"
- Write an inequality without absolute value given the above constraint;
- Sketch a graph of the inequality on the number line;
- Write an inequality using absolute value to represent the above statement;
- Repeat this process for a constraint like, "find an *x* value *at least y* units from a given number;"
- Given an inequality with absolute value, rewrite the statement without absolute value;
- Solve absolute value equations and inequalities.

I plan to use our department's old e-book for this section:

DAY 1

Suggested order of lesson:

1. Ask students to work through the "Skill prep" assignment together. This activity is available on the Blackboard site.
2. Work through the "Try Its" with your students, up to the second Checkpoint. Let students work on the checkpoints together.
3. Work through examples #1 - #5 with students, showing them the links to more worked examples.

HW due next class: **IMathAS**: HW7

DAY 2

Suggested order of lesson:

1. Work through a couple examples from the "Worked examples" worksheet on Blackboard, emphasizing that absolute value refers to distance on the number line. Students usually have no difficulty with solving absolute value equations, but have significant difficulty with the inequalities.
2. Ask students to work in pairs at the board solving absolute value inequalities OR ask students to work through #12, #7, #8 and #10 in the Pathways workbook, pp. 53-55.
3. Ask students to work on #1 and #6 on pp. 51 & 53 in the Pathways workbook.

HW due next class: **Worksheet on Blackboard: pick a selection of 10- 15 problems, asking students to make a number line sketch of their solution.** Ask them to describe in words the meaning of each problem.

REMEMBER THAT WEEKLY REVIEW ASSIGNMENTS ON IMATHAS ARE DUE SUNDAYS AT MIDNIGHT. THERE IS AN OPTIONAL PROBLEM SET ON IMATHAS WITH REVIEW PROBLEMS FOR EXAM 1.