Lecture 10.1, MATH-57091 Probability and Statistics for High-School Teachers.

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October, 27 - October 31, 2014.

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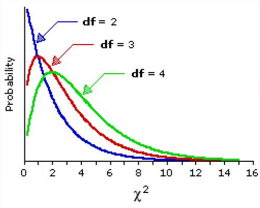

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Interval Estimators of the Mean of a Normal Population with unknown population variance

We are back to our estimators! Suppose that we have a sample $X_1, \dots, X_n$ from a (normal) population having an **unknown** mean μ and **unknown** standard deviation σ, and we want to obtain an interval estimator for μ.

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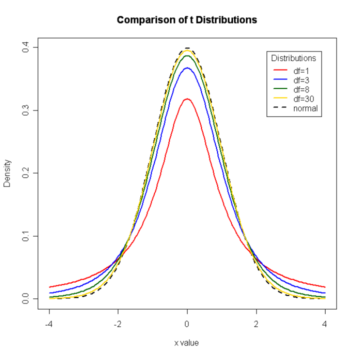

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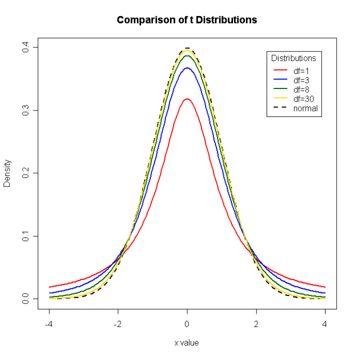

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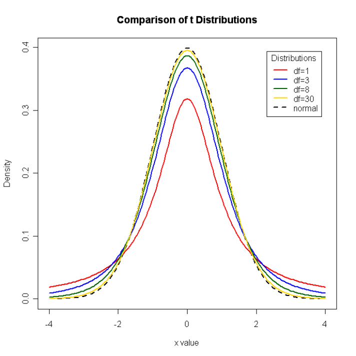

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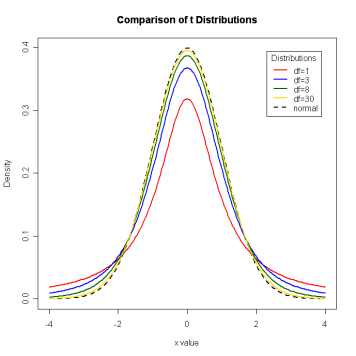

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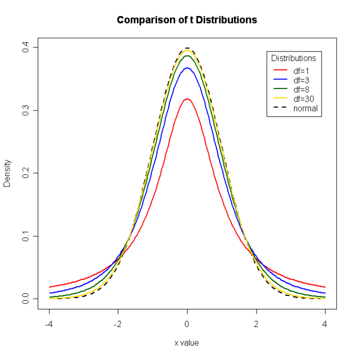

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Integration approximation and computer techniques were used to create tables that help us to evaluate (approximate) the values corresponding to t distribution with fixed degrees of freedom.

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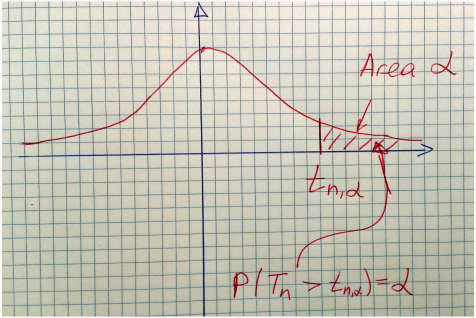

Significance level = α

Degrees of Freedom	.005 (1-tail) .01 (2-tails)	.01 (1-tail) .02 (2-tails)	.025 (1-tail) .05 (2-tails)	.05 (1-tail) .10 (2-tails)	.10 (1-tail) .20 (2-tails)	.25 (1-tail) .50 (2-tails)
1	63.657	31.821	12.706	6.314	3.078	1.000
2	9.925	6.965	4.303	2.920	1.886	.816
3	5.841	4.541	3.182	2.353	1.638	.765
4	4.604	3.747	2.776	2.132	1.533	.741
5	4.032	3.365	2.571	2.015	1.476	.727
6	3.707	3.143	2.447	1.943	1.440	.718
7	3.500	2.998	2.365	1.895	1.415	.711
8	3.355	2.896	2.306	1.860	1.397	.706
9	3.250	2.821	2.262	1.833	1.383	.703
10	3.169	2.764	2.228	1.812	1.372	.700
11	3.106	2.718	2.201	1.796	1.363	.697
12	3.054	2.681	2.179	1.782	1.356	.696
13	3.012	2.650	2.160	1.771	1.350	.694
14	2.977	2.625	2.145	1.761	1.345	.692
15	2.947	2.602	2.132	1.753	1.341	.691
16	2.921	2.584	2.120	1.746	1.337	.690
17	2.898	2.567	2.110	1.740	1.333	.689
18	2.878	2.552	2.101	1.734	1.330	.688
19	2.861	2.540	2.093	1.729	1.328	.688
20	2.845	2.528	2.086	1.725	1.325	.687
21	2.831	2.518	2.080	1.721	1.323	.686
22	2.819	2.508	2.074	1.717	1.321	.686
23	2.807	2.500	2.069	1.714	1.320	.685
24	2.797	2.492	2.064	1.711	1.318	.685
25	2.878	2.485	2.060	1.708	1.316	.684
26	2.779	2.479	2.056	1.706	1.315	.684
27	2.771	2.473	2.052	1.703	1.314	.684
28	2.763	2.467	2.048	1.701	1.313	.683
29	2.756	2.462	2.045	1.699	1.311	.683
Large	2.575	2.327	1.960	1.645	1.282	.675

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Solution: Look for 8 degrees of freedom and "1-tail" - 0.05 get $t_{8,0.05} = 1.860$.

Find $t_{6,0.005}$.

Solution: Look for 6 degrees of freedom and "1-tail" - 0.005 get $t_{6,0.005} = 3.707$.

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Symmetric Interval

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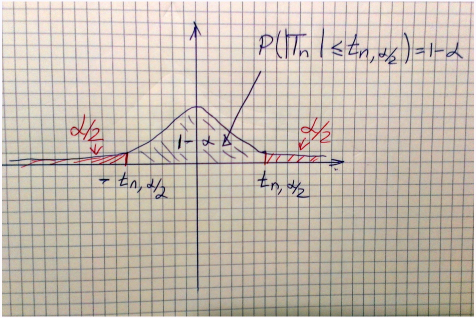

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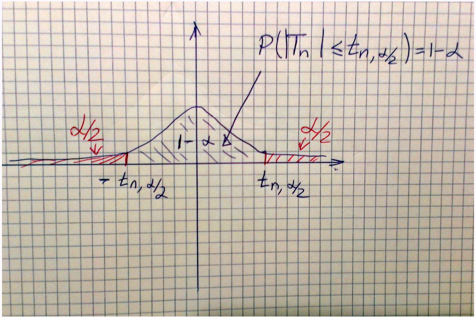

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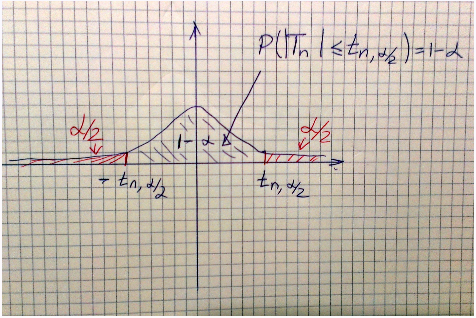

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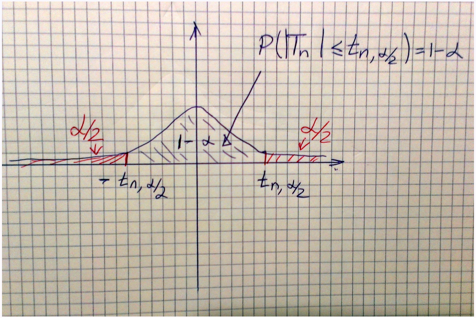

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So

$$P\left(\bar{X} - t_{n-1,\alpha/2}\frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{n-1,\alpha/2}\frac{S}{\sqrt{n}}\right) = 1 - \alpha.$$

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$$t_{19,0.05} = 1.729.$$

Thus the 95 percent upper confidence interval is

$$5.8 + 1.729 \frac{5.085}{\sqrt{20}} = 7.77.$$

That is, we can be 95 percent sure that the average PCB level will be below 7.77. We also use the table to get $t_{19,0.01} = 2.539$. Therefore the the 99 percent lower confidence interval is

$$5.8 - 2.529 \frac{5.085}{\sqrt{20}} = 2.91.$$

That is, we can be 99 percent sure that the average PCB level will be above 2.91.