

Lecture 10.2, MATH-57091 Probability and Statistics for High-School Teachers.

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Reminder: Point estimator for population proportion

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The above quantity is called the **Standard Error of \hat{p}** .

Interval Estimators of population proportion

Now we would like to provide an interval estimator for p . When n is larger enough we can use the normal approximation to the binomial distribution (please, check previous lectures) and approximate that $100(1 - \alpha)$ percent interval estimator of p is given by

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Finally

$$P\left(\left|\frac{\hat{p} - p}{SD(\hat{p})}\right| \leq z_{\alpha/2}\right) \approx P(|Z| \leq z_{\alpha/2}) = 1 - \alpha.$$

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An approximate $100(1 - \alpha)$ percent confidence interval estimator of p is given by

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}},$$

where \hat{p} is the proportion of members of the sample of size n who have the characteristic of our study.

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$$z_{\alpha/2} = z_{0.005} = 2.576.$$

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Finally, we get that with 99 percent confidence that the percentage of nonsmokers is between 72.1 and 91.9.

One more Example:

On the December 24, 1991, **The New York Times** reported that a poll indicated that 46 percent of the population of US was in favor of the way President Bush has handling the economy, with margin of error of ± 3 percent. What does this mean? Can we try to understand how many people were questioned?

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From where we get that n is about 1060 and thus among 1060 people that were asked 46 supported President Bush economy politics.

Length of the Confidence Interval

We know that $100(1 - \alpha)$ percent confidence interval for p is

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The length of a $100(1 - \alpha)$ percent confidence interval is

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Solution: 90 percent confidence interval gives $\alpha = 0.1$ and $\alpha/2 = 0.05$ and we need to work with $z_{0.05} = 1.645$ i.e.

$$n > \left(\frac{z_{0.05}}{0.01} \right)^2 = \left(\frac{1.645}{0.01} \right)^2 = 27062.25.$$

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So the sample size should be at least 27063.

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and finally, the 95 percent lower bound is given by

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Thus we can be 95 sure that over 26.6 percent of all workers are dissatisfied with their working conditions.