Lecture 11.1, MATH-57091 Probability and Statistics for High-School Teachers.

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So, how should we check such statement? Some claims may be checked precisely (just check all members of the "population") for another it is simply impossible to check it "precisely", and more probabilistic/statistical approach is required.

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The alternative to the null hypothesis is called **alternative hypothesis** and is denoted by H_1. In our example H_1 will claim that the average work life of AZ computer is less then 7 years:

$$H_1 : \mu < 7.$$

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The critical region, also called rejection region, is that set of values of the statistics for which the null hypothesis is rejected.

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where n is the size of sample.

So we need to collect random sample of size n, compute $\bar{X}$ and then our hypothesis that AZ computers can work well over 7 years is

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More precisely

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- This would let us to specify the appropriate critical region ("far away") so that probability that estimator fall in this region when H_0 is true is smaller then α.

Back to formulas: Case of known variance

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$$P(|\bar{X} - \mu_0| \geq c) = \alpha, \text{ when } \mu = \mu_0.$$

Yes, the above line is EXACTLY the statement that the chances for H_0 to be rejected (i.e. for $\bar{X} \in C$) when μ is actually equal to μ_0 is α.

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So find c we need to solve

$$P(|Z| \geq \frac{\sqrt{n}}{\sigma}c) = \alpha.$$

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$$|\bar{X} - \mu_0| \geq c \text{ is equivalent to } \frac{\sqrt{n}}{\sigma}|\bar{X} - \mu_0| \geq \frac{\sqrt{n}}{\sigma}c.$$

So find c we need to solve

$$P(|Z| \geq \frac{\sqrt{n}}{\sigma}c) = \alpha.$$

Using that Z is symmetric r.v. (density is an even function) we get

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Our goal is to select c so that

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The significance-level-α test of the null hypothesis that the population mean is equal to μ_0 against the alternative that is not equal to μ_0 is to be rejected if

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EXAMPLE!!!

Suppose that if a signal of intensity μ is emitted from a particular star, then the value received at an observatory on earth is a normal random variable with mean μ and standard deviation 4. In other words the value of the signal emitted is altered by a **random noise**, which is normally distributed with mean 0 and standard deviation 4. It is suspected that the intensity of the signal is equal to 10. Test whether this hypothesis is plausible if the same signal is independently received 20 times and the average of the 20 values received is 11.6. Use the 5 percent level of significance.

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