Lecture 11.2, MATH-57091 Probability and Statistics for High-School Teachers.

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We provide that

The significance-level-α test of the null hypothesis that the population mean is equal to μ_0 against the alternative that is not equal to μ_0 is to be rejected if

$$|\bar{X} - \mu_0| \geq \frac{\sigma}{\sqrt{n}} z_{\alpha/2}.$$

equivalently

$$\begin{array}{rl} \text{Reject } H_0 : & \frac{\sqrt{n}}{\sigma}|\bar{X} - \mu_0| \geq z_{\alpha/2} \\ \text{Not Reject } H_0 : & \frac{\sqrt{n}}{\sigma}|\bar{X} - \mu_0| < z_{\alpha/2}. \end{array}$$

EXAMPLE!!!

Suppose that if a signal of intensity μ is emitted from a particular star, then the value received at an observatory on earth is a normal random variable with mean μ and standard deviation 4. In other words the value of the signal emitted is altered by a **random noise**, which is normally distributed with mean 0 and standard deviation 4. It is suspected that the intensity of the signal is equal to 10. Test whether this hypothesis is plausible if the same signal is independently received 20 times and the average of the 20 values received is 11.6. Use the 5 percent level of significance.

Solution: If μ represents the actual intensity then the null hypothesis and alternative are

$$H_0: \quad \mu = 10$$
$$H_1: \quad \mu \neq 10$$

Thus the first step is to compute

$$\frac{\sqrt{n}}{\sigma}|\bar{X} - \mu_0| = \frac{\sqrt{20}}{4}|11.6 - 10| = 1.79$$

Next we check the tables to find out that for $\alpha = 0.05$

$$z_{\alpha/2} = z_{0.025} = 1.96 > 1.79.$$

Thus the null hypothesis is not rejected and we conclude that the data is not inconsistent with the null hypothesis. Note, that if we would take $\alpha = 0.1$, then

$$z_{\alpha/2} = z_{0.05} = 1.645 < 1.79$$

and the hypothesis would be rejected! **So the rejections or not rejection of the hypothesis do depends on the level of significance we use!** A larger significance level gives smaller $z_{\alpha/2}$ and thus more chances for hypothesis to be rejected. It is, thus logical to ask for a smallest significance level at which the data lead to rejection of H_0.

The **p value** is the smallest significance level at which the data lead to rejection of the null hypothesis. It gives the probability that data as unsupportive of H_0 as those observed will occur when H_0 is true. A small value p (less then 5 percent) is a strong indication that the null hypothesis is not true. The smaller the value of p, the larger chance that H_0 is false.

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In real life, the significance level is often not set in advance; rather the data is used to determine the p value. This value is often either so large that it is clear that the null hypothesis should not be rejected or so small that it is clear that the null hypothesis should be rejected.

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And the tables give

$$p \text{ value} = P(|Z| \geq 2.46) = 2P(Z \geq 2.46) = 0.014$$

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$$= P(Z \leq -1.066) + P(Z \geq 2.856) = 0.1432 + 0.0021 = .1453$$

Probability of not rejecting H_0, when it is false

Suppose that if a signal of intensity μ is emitted from a particular star, then the value received at an observatory on earth is a normal random variable with mean μ and standard deviation 4. In other words the value of the signal emitted is altered by a **random noise**, which is normally distributed with mean 0 and standard deviation 4. It is suspected that the intensity of the signal is equal to 10. Assume 0.05 significance level, what is the probability that the null hypothesis ($\mu = 10$) will not be rejected after measuring the sample signal 20 times if the actual value of the signal is 9.2?

Solution: So in this case $\sigma = 4$ and $n = 20$. Thus, the significance-level-0.05 test of

$$H_0 : \mu = 10 \text{ against } H_1 : \mu \neq 10$$

is rejected if

$$\frac{\sqrt{20}}{4}|\bar{X} - 10| \geq z_{0.025} \text{ which is equivalent to } |\bar{X} - 10| \geq \frac{4}{\sqrt{20}} z_{0.025}.$$

Using the tables we get $4z_{0.025}/\sqrt{20} = 1.753$. This means that H_0 is rejected if $|\bar{X} - 10| \geq 1.753$, opening the absolute value we get $\bar{X} \leq 8.247$ or $\bar{X} \geq 11.753$. So our goal is to compute the probability of the above event. We know that the population mean is 9.2, thus $\bar{X}$ is normal r.v. with mean 9.2 and standard deviation $4/\sqrt{20} = .894$, i.e. we need to work with $N(0,1)$ r.v. $Z = \frac{\bar{X} - 9.2}{0.894}$. Finally,

$$P(\text{ rejection of } H_0) = P(\bar{X} \leq 8.247) + P(\bar{X} \geq 11.753)$$

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$$= P(Z \leq -1.066) + P(Z \geq 2.856) = 0.1432 + 0.0021 = .1453$$

Thus, when the true signal value is 9.2 there is a 85.47 percent chance that 0.05 significance level test will not reject the null hypothesis $\mu = 10$.

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We will want to reject H_0 when the sample mean $\bar{X}$ is much larger than μ_0, it can be shown in exactly the same way as in two-sided case that the significance-level-α test is to

$$\begin{aligned} \text{Reject } H_0 : &\quad \frac{\sqrt{n}}{\sigma}(\bar{X} - \mu_0) \ge z_\alpha \\ \text{Not Reject } H_0 : &\quad \frac{\sqrt{n}}{\sigma}(\bar{X} - \mu_0) < z_\alpha . \end{aligned}$$

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where Z is $N(0,1)$ r.v.. The null hypothesis is then rejected at any significance level greater than or equal to the value p.

Example

All cigarettes presently being sold have an average nicotine content of at least 1.5 milligrams per cigarette, a firm is making a claim that a new technique it has is producing cigarette with nicotine level less then 1.5 milligrams. To test this claim a sample of 20 cigarettes was analyzed. If it is known that the standard deviation of a cigarette's nicotine content was .7 milligrams, what conclusion could be drawn, at the 5 percent level of significance, if the average nicotine content of these 20 cigarettes were 1.42 milligrams?

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Since the p value exceeds 0.05 the the data we have do not enable us to reject the null hypothesis! The data that we have, indeed looks strong to support the companies claim but it is not strong enough to **prove** the claim at required level!

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H_0 :	H_1	Test Statistics (TS)	Significance-level-α test	p Value if TS$=\nu$
$\mu = \mu_0$	$\mu \neq \mu_0$	$\frac{\sqrt{n}}{\sigma}(\bar{X} - \mu_0)$	Reject H_0 if $\lvert TS \rvert \geq z_{\alpha/2}$. Do not reject H_0 otherwise	$2P(Z \geq \lvert\nu\rvert)$
$\mu \leq \mu_0$	$\mu > \mu_0$	$\frac{\sqrt{n}}{\sigma}(\bar{X} - \mu_0)$	Reject H_0 if $TS \geq z_{\alpha}$. Do not reject H_0 otherwise	$P(Z \geq \nu)$
$\mu \geq \mu_0$	$\mu < \mu_0$	$\frac{\sqrt{n}}{\sigma}(\bar{X} - \mu_0)$	Reject H_0 if $TS \leq -z_{\alpha}$. Do not reject H_0 otherwise	$P(Z \leq \lvert\nu\rvert)$