Lecture 1.1, MATH-57091 Probability and Statistics for High-School Teachers.

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Instead of Introduction

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Our first goal is to give main definitions and at least try to construct some mathematical base for Probability Theory (and Statistics). We will start with some discussion of set theory.

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- $S = \{1, 2, 7\}$.

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- $S = \{6, 7, 8, \cdots\}$. Then $7 \in S$, but $1 \notin S$, $45 \in S$ and $67.5 \notin S$.

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- $S = \{n : n \text{ is natural number, which is greater then 71 and divisible by 6}\}$.

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More Definitions:

If every element of S is an element of T, then S is a *subset* of T and we denote it as $S \subset T$. If $S \subset T$ and $T \subset S$, then $S = T$.

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then

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Let A be an even that the sum of dices is less then 5. Then $A \cap T = \emptyset$ (indeed if the sum is less then 5 it can not be greater then 7).

A and T are disjoint sets if $A \cap T = \emptyset$.

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$$\bigcup_{n=1}^{\infty} S_n = \{x : x \in S_n, \text{ for at least one } n\}$$

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Example:

Let $\mathbb{R}$ be a set of real numbers and let $S_n = \{x \in \mathbb{R} : |x| \leq n\}$, then $\bigcup_{n=1}^{\infty} S_n = \mathbb{R}$ and $\bigcap_{n=1}^{\infty} S_n = [-1, 1]$.

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3. $S \cap (T \cup U) = (T \cap S) \cup (S \cap U)$.

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2. $S \cup T = T \cup S$.
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4. $(S^c)^c = S$.

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For example, $(A \cap B)^c = A^c \cup B^c$.

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Which is equivalent to say that $x \in \Omega$ and $x \in S_n^c$ for all n. Which is exactly $x \in \bigcap_n S_n^c$. So we are done with the first inclusion!

Writing, the above logical statements in reverse direction we get the second inclusion. More precisely, assume that $x \in \bigcap_n S_n^c$, then $x \in S_n^c$ for all n. Which is equivalent to say that $x \in \Omega$ but $x \notin S_n$ for all n. But then $x \in \Omega$ and $x \notin \bigcup_n S_n$, which is exactly the same as $x \in \left(\bigcup_n S_n\right)^c$ and we are finished with second inclusion.