Lecture 2.2,
MATH-57091 Probability and Statistics for High-School Teachers.

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Thus A_1 and B are dependent events! So the event of the sum of two dices to be 6 really depends on what is the first dice (yes! it is good if the first one is $1,2,3,4,5$ we just need the second do match it, which will happened with probability $1/6$, but if the first one is 6 then we are out of any luck there is no way for the second one to match it and to make sum to be 6!

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and (please, check the table in previous lectures)

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Thus we note that A_2 and B are independent! An intuitive way to see it is to notice that we actually do not care much in this case what is the first dice! The probability for the second one to match it is always $1/6$.

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$$\begin{aligned} P(A) &= P((A \cap B) \cup (A \cap B^c)) \\ &= P(A \cap B) + P(A \cap B^c) \\ &= P(A|B)P(B) + P(A|B^c)P(B^c) \end{aligned}$$

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Intuitively the above formula tell us that probability of event A is a weighted sum of the conditional probability of A given B and conditional probability of A given B^c, where each conditional probability is given as much weight as the event it is conditioned on has probability of occurring.

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Problem: What is the probability that the ball was taken from the first urn if we know that the ball is green?
Solution: We are looking for $P(F|G)$ we notice that

$$P(F|G) = \frac{P(F \cap G)}{P(G)} = \frac{P(F \cap G)P(F)}{P(F)P(G)} = P(G|F)\frac{P(F)}{P(G)} = \frac{3}{5} \times \frac{1/2}{.6125} = \frac{.3}{.6125}.$$

Problem: In answering a question on a multiple-choice yes a student either knows the answer or guesses. Let p be the probability that she knows the answer and $1 - p$ the probability that she guesses. Assume that a student who guesses the answer will be correct with probability $1/m$ (where m is a number of multiple-choice alternatives). What is the conditional probability that a student knew the answer to a question given that she answered it correctly?

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$$\begin{aligned} P(K|C) &= \frac{P(C|K)P(K)}{P(C)} \\ &= \frac{P(C|K)P(K)}{P(C|K)P(K)+P(C|K^c)P(K^c)} \\ &= \frac{1*p}{1*p+\frac{1}{m}(1-p)}. \end{aligned}$$