Lecture 3.2, MATH-57091 Probability and Statistics for High-School Teachers.

Artem Zvavitch

Department of Mathematical Sciences, Kent State University

September, 8 - 12, 2014.

A random variable that can take at most a countable (finite) set of possible values is said to be **discrete**.

Discrete Random Variables

A random variable that can take at most a countable (finite) set of possible values is said to be **discrete**. In this case we may define a probability mass function, i.e. the probability that a discrete random variable is exactly equal to some value.

A random variable that can take at most a countable (finite) set of possible values is said to be **discrete**. In this case we may define a probability mass function, i.e. the probability that a discrete random variable is exactly equal to some value. More precisely X takes value $x_1, x_2, \dots$ such that

$$p(x_i) = P(X = x_i),$$

where

$$p(x_i) \in [0, 1] \text{ and } \sum_{i=1}^{\infty} p(x_i) = 1.$$

A random variable that can take at most a countable (finite) set of possible values is said to be **discrete**. In this case we may define a probability mass function, i.e. the probability that a discrete random variable is exactly equal to some value. More precisely X takes value $x_1, x_2, \dots$ such that

$$p(x_i) = P(X = x_i),$$

where

$$p(x_i) \in [0,1] \text{ and } \sum_{i=1}^{\infty} p(x_i) = 1.$$

If X is a discrete Random variable that take one of the possible values $x_1, x_2, \dots, x_n$ then the **expected value** of X, denoted by $\mathbb{E}X$ is defined by

$$\mathbb{E}X = \sum_{i=1}^{n} x_i P(X = x_i) = \sum x_i p(x_i).$$

Bernoulli Random Variable

Suppose that a trial or an experiment, whose outcome can be classified as either a "success" or as "failure", is performed.

Suppose that a trial or an experiment, whose outcome can be classified as either a "success" or as "failure", is performed. If we let X equal 1 if the outcome us a success and 0 if it is a failure then the probability mass function is given by

$$p(0) = P(X = 0) = 1 - p,$$

$$p(1) = P(X = 1) = p,$$

where $p \in [0, 1]$ is a probability that the trial is a "success".

Suppose that a trial or an experiment, whose outcome can be classified as either a "success" or as "failure", is performed. If we let X equal 1 if the outcome us a success and 0 if it is a failure then the probability mass function is given by

$$p(0) = P(X = 0) = 1 - p,$$

$$p(1) = P(X = 1) = p,$$

where $p \in [0,1]$ is a probability that the trial is a "success". A random variable X is said to be a **Bernoulli random variable** if its as above and $p \in (0,1)$.

Suppose that a trial or an experiment, whose outcome can be classified as either a "success" or as "failure", is performed. If we let X equal 1 if the outcome us a success and 0 if it is a failure then the probability mass function is given by

$$p(0) = P(X = 0) = 1 - p,$$

$$p(1) = P(X = 1) = p,$$

where $p \in [0,1]$ is a probability that the trial is a "success". A random variable X is said to be a **Bernoulli random variable** if its as above and $p \in (0,1)$.

Expectation of Bernoulli Random Variable

It is not so hard to compute $\mathbb{E}X$ if X is a Bernoulli Random variable with parameter p:

$$\mathbb{E}X = 0 * (1-p) + 1 * p = p.$$

The Binomial Random Variable

Suppose that n independent trial, each of which results in a "success" with probability p and "failure" with probability $1 - p$ are to be performed.

The Binomial Random Variable

Suppose that n independent trial, each of which results in a "success" with probability p and "failure" with probability $1 - p$ are to be performed. If X represents the number of successes that occur in the n trials, then X is said to be a **Binomial Random Variable** with parameters (n, p).

The Binomial Random Variable

Suppose that n independent trial, each of which results in a "success" with probability p and "failure" with probability $1-p$ are to be performed. If X represents the number of successes that occur in the n trials, then X is said to be a **Binomial Random Variable** with parameters (n,p). We note that

$$p(i) = P(X=i) = \binom{n}{i} p^i (1-p)^{n-i}, \text{ for } i = 0, 1, \ldots, n.$$

where

$$\binom{n}{i} = \frac{n!}{(n-i)!i!}.$$

The Binomial Random Variable

Suppose that n independent trial, each of which results in a "success" with probability p and "failure" with probability $1-p$ are to be performed. If X represents the number of successes that occur in the n trials, then X is said to be a **Binomial Random Variable** with parameters (n,p). We note that

$$p(i) = P(X=i) = \binom{n}{i} p^i (1-p)^{n-i}, \text{ for } i = 0, 1, \ldots, n.$$

where

$$\binom{n}{i} = \frac{n!}{(n-i)!i!}.$$

Please, note that the above formula is not so hard to verify! First notice that for particular sequence of n outcomes to have exactly i successes and thus $n-i$ failures is exactly $p^i(1-p)^{n-i}$. But we have a lot of ways to select "when" our i successes occur and the number of those "ways" is exactly $\binom{n}{i}$.

Suppose that n independent trial, each of which results in a "success" with probability p and "failure" with probability $1-p$ are to be performed. If X represents the number of successes that occur in the n trials, then X is said to be a **Binomial Random Variable** with parameters (n,p). We note that

$$p(i) = P(X = i) = \binom{n}{i} p^i (1-p)^{n-i}, \text{ for } i = 0, 1, \ldots, n.$$

where

$$\binom{n}{i} = \frac{n!}{(n-i)!i!}.$$

Please, note that the above formula is not so hard to verify! First notice that for particular sequence of n outcomes to have exactly i successes and thus $n-i$ failures is exactly $p^i(1-p)^{n-i}$. But we have a lot of ways to select "when" our i successes occur and the number of those "ways" is exactly $\binom{n}{i}$. It is also a good moment to remind the Binomial Formula

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}.$$

Suppose that n independent trial, each of which results in a "success" with probability p and "failure" with probability $1-p$ are to be performed. If X represents the number of successes that occur in the n trials, then X is said to be a **Binomial Random Variable** with parameters (n,p). We note that

$$p(i)=P(X=i)=\binom{n}{i}p^i(1-p)^{n-i}, \text{ for } i=0,1,\dots,n.$$

where

$$\binom{n}{i}=\frac{n!}{(n-i)!i!}.$$

Please, note that the above formula is not so hard to verify! First notice that for particular sequence of n outcomes to have exactly i successes and thus $n-i$ failures is exactly $p^i(1-p)^{n-i}$. But we have a lot of ways to select "when" our i successes occur and the number of those "ways" is exactly $\binom{n}{i}$. It is also a good moment to remind the Binomial Formula

$$(x+y)^n=\sum_{i=0}^{n}\binom{n}{i}x^iy^{n-i}.$$

One can see the formula just from noticing that $(x+y)^n$ is $(x+y)$ multiplied n times. Thus when we open the parentheses you need to select x or y from each multiplier only once so to get x^iy^{n-i} you need to pick i - x and $(n-i)$ - y and you can do it in $\binom{n}{i}$ ways.

Suppose that n independent trial, each of which results in a "success" with probability p and "failure" with probability $1-p$ are to be performed. If X represents the number of successes that occur in the n trials, then X is said to be a **Binomial Random Variable** with parameters (n,p). We note that

$$p(i) = P(X=i) = \binom{n}{i} p^i (1-p)^{n-i}, \text{ for } i = 0, 1, \dots, n.$$

where

$$\binom{n}{i} = \frac{n!}{(n-i)!i!}.$$

Please, note that the above formula is not so hard to verify! First notice that for particular sequence of n outcomes to have exactly i successes and thus $n-i$ failures is exactly $p^i(1-p)^{n-i}$. But we have a lot of ways to select "when" our i successes occur and the number of those "ways" is exactly $\binom{n}{i}$. It is also a good moment to remind the Binomial Formula

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}.$$

One can see the formula just from noticing that $(x+y)^n$ is $(x+y)$ multiplied n times. Thus when we open the parentheses you need to select x or y from each multiplier only once so to get $x^i y^{n-i}$ you need to pick i - x and $(n-i)$ - y and you can do it in $\binom{n}{i}$ ways. The binomial formula immediately gives that

$$\sum_{i=0}^{n} P(X=i) = \sum_{i=0}^{n} \binom{n}{i} p^i (1-p)^{n-i} = (p+(1-p))^n = 1.$$

Questions: Four fair coins are flipped. If the outcomes are assumes independent, what is the probability that two heads and two tails are obtained?

Questions: Four fair coins are flipped. If the outcomes are assumes independent, what is the probability that two heads and two tails are obtained? **Solution:** Letting X equal the number of heads ("successes") that appear, then X is a binomial random variable with parameters $n = 4, p = \frac{1}{2}$.

Questions: Four fair coins are flipped. If the outcomes are assumes independent, what is the probability that two heads and two tails are obtained? **Solution:** Letting X equal the number of heads ("successes") that appear, then X is a binomial random variable with parameters $n = 4, p = \frac{1}{2}$. So

$$P(X = 2) = \binom{4}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = \frac{3}{8}.$$

Questions: Four fair coins are flipped. If the outcomes are assumes independent, what is the probability that two heads and two tails are obtained? **Solution:** Letting X equal the number of heads ("successes") that appear, then X is a binomial random variable with parameters $n = 4, p = \frac{1}{2}$. So

$$P(X = 2) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{3}{8}.$$

Questions: It is known that all computers produced by BRC will be defective with probability .2, independently of each other. What is the probability that in a sample of four computers, at most 1 is defective?

Questions: Four fair coins are flipped. If the outcomes are assumes independent, what is the probability that two heads and two tails are obtained? **Solution:** Letting X equal the number of heads ("successes") that appear, then X is a binomial random variable with parameters $n = 4, p = \frac{1}{2}$. So

$$P(X = 2) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{3}{8}.$$

Questions: It is known that all computers produced by BRC will be defective with probability .2, independently of each other. What is the probability that in a sample of four computers, at most 1 is defective?
Solution: Let X be a number of defective items in the sample, then X is a binomial random variable with parameters $(4, .2)$.

The Binomial Random Variable: Examples

Questions: Four fair coins are flipped. If the outcomes are assumes independent, what is the probability that two heads and two tails are obtained? **Solution:** Letting X equal the number of heads ("successes") that appear, then X is a binomial random variable with parameters $n=4, p=\frac{1}{2}$. So

$$P(X=2)=\binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2=\frac{3}{8}.$$

Questions: It is known that all computers produced by BRC will be defective with probability .2, independently of each other. What is the probability that in a sample of four computers, at most 1 is defective?
Solution: Let X be a number of defective items in the sample, then X is a binomial random variable with parameters $(4, .2)$. Thus, the desired probability is given by

$$P(X=0)+P(X=1)=\binom{4}{0}.2^0.8^4+\binom{4}{1}.2^1.8^3=.8192$$

The Binomial Random Variable: Expectation

Let X be a binomial random variable with parameters (n, p) then

The Binomial Random Variable: Expectation

Let X be a binomial random variable with parameters (n, p) then

$$\mathbb{E}X = \sum_{i=0}^{n} i \binom{n}{i} p^i (1-p)^{n-i} = \sum_{i=1}^{n} i \frac{n!}{(n-i)!i!} p^i (1-p)^{n-i}$$

Let X be a binomial random variable with parameters (n, p) then

$$\mathbb{E}X = \sum_{i=0}^{n} i \binom{n}{i} p^i (1-p)^{n-i} = \sum_{i=1}^{n} i \frac{n!}{(n-i)!i!} p^i (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} \frac{n!}{(n-i)!(i-1)!} p^i (1-p)^{n-i}$$

Let X be a binomial random variable with parameters (n, p) then

$$\mathbb{E}X = \sum_{i=0}^{n} i \binom{n}{i} p^i (1-p)^{n-i} = \sum_{i=1}^{n} i \frac{n!}{(n-i)!i!} p^i (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} \frac{n!}{(n-i)!(i-1)!} p^i (1-p)^{n-i}$$

$$= np \sum_{i=1}^{n} \frac{(n-1)!}{((n-1)-(i-1))!(i-1)!} p^{i-1} (1-p)^{(n-1)-(i-1)}$$

The Binomial Random Variable: Expectation

Let X be a binomial random variable with parameters (n, p) then

$$\mathbb{E}X = \sum_{i=0}^{n} i \binom{n}{i} p^i (1-p)^{n-i} = \sum_{i=1}^{n} i \frac{n!}{(n-i)!i!} p^i (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} \frac{n!}{(n-i)!(i-1)!} p^i (1-p)^{n-i}$$

$$= np \sum_{i=1}^{n} \frac{(n-1)!}{((n-1)-(i-1))!(i-1)!} p^{i-1} (1-p)^{(n-1)-(i-1)}$$

$$= np \sum_{j=0}^{n-1} \frac{(n-1)!}{((n-1)-j)!j!} p^j (1-p)^{(n-1)-j}$$

The Binomial Random Variable: Expectation

Let X be a binomial random variable with parameters (n, p) then

$$\mathbb{E}X = \sum_{i=0}^{n} i \binom{n}{i} p^i (1-p)^{n-i} = \sum_{i=1}^{n} i \frac{n!}{(n-i)!i!} p^i (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} \frac{n!}{(n-i)!(i-1)!} p^i (1-p)^{n-i}$$

$$= np \sum_{i=1}^{n} \frac{(n-1)!}{((n-1)-(i-1))!(i-1)!} p^{i-1} (1-p)^{(n-1)-(i-1)}$$

$$= np \sum_{j=0}^{n-1} \frac{(n-1)!}{((n-1)-j)!j!} p^j (1-p)^{(n-1)-j}$$

$$= np \sum_{j=0}^{n-1} \binom{n-1}{j} p^j (1-p)^{(n-1)-j}$$

Let X be a binomial random variable with parameters (n, p) then

$$\mathbb{E}X = \sum_{i=0}^{n} i \binom{n}{i} p^i (1-p)^{n-i} = \sum_{i=1}^{n} i \frac{n!}{(n-i)!i!} p^i (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} \frac{n!}{(n-i)!(i-1)!} p^i (1-p)^{n-i}$$

$$= np \sum_{i=1}^{n} \frac{(n-1)!}{((n-1)-(i-1))!(i-1)!} p^{i-1} (1-p)^{(n-1)-(i-1)}$$

$$= np \sum_{j=0}^{n-1} \frac{(n-1)!}{((n-1)-j)!j!} p^j (1-p)^{(n-1)-j}$$

$$= np \sum_{j=0}^{n-1} \binom{(n-1)}{j} p^j (1-p)^{(n-1)-j}$$

$$= np(p + (1-p))^{(n-1)}$$

The Binomial Random Variable: Expectation

Let X be a binomial random variable with parameters (n, p) then

$$\mathbb{E}X = \sum_{i=0}^{n} i\binom{n}{i} p^i (1-p)^{n-i} = \sum_{i=1}^{n} i\frac{n!}{(n-i)!i!} p^i (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} \frac{n!}{(n-i)!(i-1)!} p^i (1-p)^{n-i}$$

$$= np\sum_{i=1}^{n} \frac{(n-1)!}{((n-1)-(i-1))!(i-1)!} p^{i-1} (1-p)^{(n-1)-(i-1)}$$

$$= np\sum_{j=0}^{n-1} \frac{(n-1)!}{((n-1)-j)!j!} p^j (1-p)^{(n-1)-j}$$

$$= np\sum_{j=0}^{n-1} \binom{n-1}{j} p^j (1-p)^{(n-1)-j}$$

$$= np(p+(1-p))^{(n-1)}$$

$$= np$$

The Binomial Random Variable: Expectation

Let X be a binomial random variable with parameters (n, p) then

$$\mathbb{E}X = \sum_{i=0}^{n} i \binom{n}{i} p^i (1-p)^{n-i} = \sum_{i=1}^{n} i \frac{n!}{(n-i)!i!} p^i (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} \frac{n!}{(n-i)!(i-1)!} p^i (1-p)^{n-i}$$

$$= np \sum_{i=1}^{n} \frac{(n-1)!}{((n-1)-(i-1))!(i-1)!} p^{i-1} (1-p)^{(n-1)-(i-1)}$$

$$= np \sum_{j=0}^{n-1} \frac{(n-1)!}{((n-1)-j)!j!} p^j (1-p)^{(n-1)-j}$$

$$= np \sum_{j=0}^{n-1} \binom{n-1}{j} p^j (1-p)^{(n-1)-j}$$

$$= np(p + (1-p))^{(n-1)}$$

$$= np$$

The above is fun combinatorial way to compute expectation of Binomial Random Variable...soon

Suppose that independent trial, each of which results in a "success" with probability p and "failure" with probability $1-p$ are to be performed until success occurs.

Suppose that independent trial, each of which results in a "success" with probability p and "failure" with probability $1-p$ are to be performed until success occurs. If X represents the number of trial required until the first success, then X is said to be a **Geometric Random Variable** with parameters p.

Suppose that independent trial, each of which results in a "success" with probability p and "failure" with probability $1-p$ are to be performed until success occurs. If X represents the number of trial required until the first success, then X is said to be a **Geometric Random Variable** with parameters p. We note that

$$p(n) = P(X = n) = (1-p)^{n-1}p, \text{ for } n = 1, 2, 3, \ldots,$$

(please, explain why!).

Suppose that independent trial, each of which results in a "success" with probability p and "failure" with probability $1-p$ are to be performed until success occurs. If X represents the number of trial required until the first success, then X is said to be a **Geometric Random Variable** with parameters p. We note that

$$p(n) = P(X = n) = (1-p)^{n-1}p, \text{ for } n = 1, 2, 3, \ldots,$$

(please, explain why!).

Please, note that:

$$\sum_{n=1}^{\infty} p(n) = \sum_{n=1}^{\infty} (1-p)^{n-1}p = p\sum_{n=1}^{\infty}(1-p)^{n-1}$$

Suppose that independent trial, each of which results in a "success" with probability p and "failure" with probability $1-p$ are to be performed until success occurs. If X represents the number of trial required until the first success, then X is said to be a **Geometric Random Variable** with parameters p. We note that

$$p(n) = P(X = n) = (1-p)^{n-1}p, \text{ for } n = 1, 2, 3, \ldots,$$

(please, explain why!).

Please, note that:

$$\sum_{n=1}^{\infty} p(n) = \sum_{n=1}^{\infty} (1-p)^{n-1}p = p\sum_{n=1}^{\infty}(1-p)^{n-1}$$

$$= p\sum_{n=0}^{\infty}(1-p)^{n} = p\frac{1}{1-(1-p)} = 1$$

Expectation of Geometric Random Variable

To compute $\mathbb{E}X$ for geometric random variable X one need to use quite a nice analytic trick.

Expectation of Geometric Random Variable

To compute $\mathbb{E}X$ for geometric random variable X one need to use quite a nice analytic trick. First we again remind the formula of the sum of geometric progression

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \text{ for } x \in (0,1).$$

Expectation of Geometric Random Variable

To compute $\mathbb{E}X$ for geometric random variable X one need to use quite a nice analytic trick. First we again remind the formula of the sum of geometric progression

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \text{ for } x \in (0,1).$$

Then taking the derivatives from both sides we get

$$\frac{d}{dx}\sum_{n=0}^{\infty} x^n = \frac{d}{dx}\frac{1}{1-x}, \text{ for } x \in (0,1).$$

Expectation of Geometric Random Variable

To compute $\mathbb{E}X$ for geometric random variable X one need to use quite a nice analytic trick. First we again remind the formula of the sum of geometric progression

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \text{ for } x \in (0,1).$$

Then taking the derivatives from both sides we get

$$\frac{d}{dx}\sum_{n=0}^{\infty} x^n = \frac{d}{dx}\frac{1}{1-x}, \text{ for } x \in (0,1).$$

$$\sum_{n=0}^{\infty} \frac{d}{dx} x^n = \frac{d}{dx}\frac{1}{1-x}, \text{ for } x \in (0,1).$$

Expectation of Geometric Random Variable

To compute $\mathbb{E}X$ for geometric random variable X one need to use quite a nice analytic trick. First we again remind the formula of the sum of geometric progression

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \text{ for } x \in (0,1).$$

Then taking the derivatives from both sides we get

$$\frac{d}{dx}\sum_{n=0}^{\infty} x^n = \frac{d}{dx}\frac{1}{1-x}, \text{ for } x \in (0,1).$$

$$\sum_{n=0}^{\infty} \frac{d}{dx}x^n = \frac{d}{dx}\frac{1}{1-x}, \text{ for } x \in (0,1).$$

$$\sum_{n=1}^{\infty} nx^{n-1} = \frac{1}{(1-x)^2}, \text{ for } x \in (0,1).$$

Expectation of Geometric Random Variable

To compute $\mathbb{E}X$ for geometric random variable X one need to use quite a nice analytic trick. First we again remind the formula of the sum of geometric progression

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \text{ for } x \in (0,1).$$

Then taking the derivatives from both sides we get

$$\frac{d}{dx} \sum_{n=0}^{\infty} x^n = \frac{d}{dx} \frac{1}{1-x}, \text{ for } x \in (0,1).$$

$$\sum_{n=0}^{\infty} \frac{d}{dx} x^n = \frac{d}{dx} \frac{1}{1-x}, \text{ for } x \in (0,1).$$

$$\sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}, \text{ for } x \in (0,1).$$

Then

$$\mathbb{E}X = \sum_{n=1}^{\infty} n(1-p)^{n-1} p = p \sum_{n=1}^{\infty} n(1-p)^{n-1} = p \frac{1}{(1-(1-p))^2} = \frac{1}{p}$$

Functions of a Random Variable

Random Variable is function from Sample Space to the real line: so you can easily combine it with other functions from real line to the real line.

Functions of a Random Variable

Random Variable is function from Sample Space to the real line: so you can easily combine it with other functions from real line to the real line.

Consider a Bernoulli Random Variable X:

$$P(X=0)=1-p \text{ and } P(X=1)=p,$$

and consider a function $f(x)=3x+5$,

Random Variable is function from Sample Space to the real line: so you can easily combine it with other functions from real line to the real line.

Consider a Bernoulli Random Variable X:

$$P(X=0)=1-p \text{ and } P(X=1)=p,$$

and consider a function $f(x)=3x+5$, notice that $f(0)=5$ and $f(1)=8$,

Functions of a Random Variable

Random Variable is function from Sample Space to the real line: so you can easily combine it with other functions from real line to the real line.

Consider a Bernoulli Random Variable X:

$$P(X=0)=1-p \text{ and } P(X=1)=p,$$

and consider a function $f(x)=3x+5$, notice that $f(0)=5$ and $f(1)=8$, so $f(X)$ is a new random variables taking values 5 and 8 such that

Random Variable is function from Sample Space to the real line: so you can easily combine it with other functions from real line to the real line.

Consider a Bernoulli Random Variable X:

$$P(X=0)=1-p \text{ and } P(X=1)=p,$$

and consider a function $f(x)=3x+5$, notice that $f(0)=5$ and $f(1)=8$, so $f(X)$ is a new random variables taking values 5 and 8 such that

$$P(f(X)=5)=1-p \text{ and } P(f(X)=8)=p.$$

Random Variable is function from Sample Space to the real line: so you can easily combine it with other functions from real line to the real line.

Consider a Bernoulli Random Variable X:

$$P(X=0)=1-p \text{ and } P(X=1)=p,$$

and consider a function $f(x)=3x+5$, notice that $f(0)=5$ and $f(1)=8$, so $f(X)$ is a new random variables taking values 5 and 8 such that

$$P(f(X)=5)=1-p \text{ and } P(f(X)=8)=p.$$

BE CAREFUL

Consider a random variable Y, taking values $0,1,2$ such that

$$P(Y=0)=.2, P(Y=1)=.3 \text{ and } P(Y=2)=.5.$$

Random Variable is function from Sample Space to the real line: so you can easily combine it with other functions from real line to the real line.

Consider a Bernoulli Random Variable X:

$$P(X=0)=1-p \text{ and } P(X=1)=p,$$

and consider a function $f(x)=3x+5$, notice that $f(0)=5$ and $f(1)=8$, so $f(X)$ is a new random variables taking values 5 and 8 such that

$$P(f(X)=5)=1-p \text{ and } P(f(X)=8)=p.$$

BE CAREFUL

Consider a random variable Y, taking values $0,1,2$ such that

$$P(Y=0)=.2, P(Y=1)=.3 \text{ and } P(Y=2)=.5.$$

Consider function $g(x)=x^2-x+5$,

Random Variable is function from Sample Space to the real line: so you can easily combine it with other functions from real line to the real line.

Consider a Bernoulli Random Variable X:

$$P(X=0)=1-p \text{ and } P(X=1)=p,$$

and consider a function $f(x)=3x+5$, notice that $f(0)=5$ and $f(1)=8$, so $f(X)$ is a new random variables taking values 5 and 8 such that

$$P(f(X)=5)=1-p \text{ and } P(f(X)=8)=p.$$

BE CAREFUL

Consider a random variable Y, taking values $0,1,2$ such that

$$P(Y=0)=.2, P(Y=1)=.3 \text{ and } P(Y=2)=.5.$$

Consider function $g(x)=x^2-x+5$, then $g(0)=5$, $g(1)=5$ and $g(2)=7$

Random Variable is function from Sample Space to the real line: so you can easily combine it with other functions from real line to the real line.

Consider a Bernoulli Random Variable X:

$$P(X=0)=1-p \text{ and } P(X=1)=p,$$

and consider a function $f(x)=3x+5$, notice that $f(0)=5$ and $f(1)=8$, so $f(X)$ is a new random variables taking values 5 and 8 such that

$$P(f(X)=5)=1-p \text{ and } P(f(X)=8)=p.$$

BE CAREFUL

Consider a random variable Y, taking values $0,1,2$ such that

$$P(Y=0)=.2, P(Y=1)=.3 \text{ and } P(Y=2)=.5.$$

Consider function $g(x)=x^2-x+5$, then $g(0)=5$, $g(1)=5$ and $g(2)=7$ thus $g(Y)$ is a new random variable which takes values 5 and 7 and

Random Variable is function from Sample Space to the real line: so you can easily combine it with other functions from real line to the real line.

Consider a Bernoulli Random Variable X:

$$P(X=0)=1-p \text{ and } P(X=1)=p,$$

and consider a function $f(x)=3x+5$, notice that $f(0)=5$ and $f(1)=8$, so $f(X)$ is a new random variables taking values 5 and 8 such that

$$P(f(X)=5)=1-p \text{ and } P(f(X)=8)=p.$$

BE CAREFUL

Consider a random variable Y, taking values $0,1,2$ such that

$$P(Y=0)=.2, P(Y=1)=.3 \text{ and } P(Y=2)=.5.$$

Consider function $g(x)=x^2-x+5$, then $g(0)=5$, $g(1)=5$ and $g(2)=7$ thus $g(Y)$ is a new random variable which takes values 5 and 7 and

$$P(g(Y)=5)=P(Y=0)+P(Y=1)=.2+.3=.5,$$

Random Variable is function from Sample Space to the real line: so you can easily combine it with other functions from real line to the real line.

Consider a Bernoulli Random Variable X:

$$P(X=0)=1-p \text{ and } P(X=1)=p,$$

and consider a function $f(x)=3x+5$, notice that $f(0)=5$ and $f(1)=8$, so $f(X)$ is a new random variables taking values 5 and 8 such that

$$P(f(X)=5)=1-p \text{ and } P(f(X)=8)=p.$$

BE CAREFUL

Consider a random variable Y, taking values $0,1,2$ such that

$$P(Y=0)=.2, P(Y=1)=.3 \text{ and } P(Y=2)=.5.$$

Consider function $g(x)=x^2-x+5$, then $g(0)=5$, $g(1)=5$ and $g(2)=7$ thus $g(Y)$ is a new random variable which takes values 5 and 7 and

$$P(g(Y)=5)=P(Y=0)+P(Y=1)=.2+.3=.5,$$

$$P(g(Y)=7)=.5.$$

Theorem:

Let X be a discrete random variable and a, b are constants, then

$$\mathbb{E}(aX + b) = a\mathbb{E}(X) + b$$

Theorem:

Let X be a discrete random variable and a, b are constants, then

$$\mathbb{E}(aX+b) = a\mathbb{E}(X)+b$$

Proof:

$$\mathbb{E}(aX+b) = \sum_{x:p(x)>0}(ax_i+b)p(x_i) = a\left(\sum_{x:p(x)>0}x_i p(x_i)\right) + b\sum_{x:p(x)>0}p(x_i) = a\mathbb{E}(X)+b,$$

where we used that $\sum_{x:p(x)>0} p(x_i) = 1.$

Theorem:

Let X be a discrete random variable and a, b are constants, then

$$\mathbb{E}(aX+b) = a\mathbb{E}(X)+b$$

Proof:

$$\mathbb{E}(aX+b) = \sum_{x:p(x)>0}(ax_i+b)p(x_i) = a\left(\sum_{x:p(x)>0} x_i p(x_i)\right) + b\sum_{x:p(x)>0} p(x_i) = a\mathbb{E}(X)+b,$$

where we used that $\sum_{x:p(x)>0} p(x_i) = 1.$

Find $\mathbb{E}Y$ if $Y = 3X - 7$, where X is a binomial random variables with parameters $(5, .3)$

Expectation of Functions of a Random Variable

Theorem:

Let X be a discrete random variable and a, b are constants, then

$$\mathbb{E}(aX+b) = a\mathbb{E}(X) + b$$

Proof:

$$\mathbb{E}(aX+b) = \sum_{x:p(x)>0} (ax_i+b)p(x_i) = a\left(\sum_{x:p(x)>0} x_i p(x_i)\right) + b\sum_{x:p(x)>0} p(x_i) = a\mathbb{E}(X)+b,$$

where we used that $\sum_{x:p(x)>0} p(x_i) = 1.$

Find $\mathbb{E}Y$ if $Y = 3X-7$, where X is a binomial random variables with parameters $(5, .3)$

Solution: Using the formula for expectation of Binomial random variable we get $\mathbb{E}X = 1.5$ and thus

$$\mathbb{E}Y = \mathbb{E}(3X-7) = 3\mathbb{E}X - 7 = 3 * 1.5 - 7 = -2.5.$$