

Lecture 9.1, MATH-57091 Probability and Statistics for High-School Teachers.

Artem Zvavitch

Department of Mathematical Sciences, Kent State University

October, 20 - October 24, 2014.

In previous lectures we estimated the parameter by a point estimator (for example population proportion p by point estimator \hat{p}).

In previous lectures we estimated the parameter by a point estimator (for example population proportion p by point estimator \hat{p}). Clearly, we do not expect the resulting estimator to be exactly equal to the parameter,

In previous lectures we estimated the parameter by a point estimator (for example population proportion p by point estimator \hat{p}). Clearly, we do not expect the resulting estimator to be exactly equal to the parameter, but we do expect it to be reasonably close. To make "reasonably close" more reasonable and mathematical we sometimes try to find an interval surrounding the estimator in which we can be highly confident that the parameter lies:

In previous lectures we estimated the parameter by a point estimator (for example population proportion p by point estimator \hat{p}). Clearly, we do not expect the resulting estimator to be exactly equal to the parameter, but we do expect it to be reasonably close. To make "reasonably close" more reasonable and mathematical we sometimes try to find an interval surrounding the estimator in which we can be highly confident that the parameter lies:

INTERVAL ESTIMATOR

An **interval estimator** of a population parameter is an interval that is predicted to contain the parameter. The confidence we assign to the interval is the probability that it will contain the parameter.

In previous lectures we estimated the parameter by a point estimator (for example population proportion p by point estimator \hat{p}). Clearly, we do not expect the resulting estimator to be exactly equal to the parameter, but we do expect it to be reasonably close. To make "reasonably close" more reasonable and mathematical we sometimes try to find an interval surrounding the estimator in which we can be highly confident that the parameter lies:

INTERVAL ESTIMATOR

An **interval estimator** of a population parameter is an interval that is predicted to contain the parameter. The confidence we assign to the interval is the probability that it will contain the parameter.

To determine an interval estimator of a population parameter we use the probability distribution (or approximation) of the point estimator of that parameter. We will start with the case of the interval estimator of the normal mean μ , we will assume (for now!) that the population standard deviation is known.

Let X_1, X_2, \dots be a sequence of independent random variables having common distribution and let $\mathbb{E}X_i = \mu$ and $\text{Var}(X_i) = \sigma^2$. Then, the distribution of

$$\frac{X_1 + X_2 + \dots + X_n - n\mu}{\sigma\sqrt{n}}$$

tends to the standard normal distribution Z as $n \rightarrow \infty$. That is

$$P\left(\frac{X_1 + X_2 + \dots + X_n - n\mu}{\sigma\sqrt{n}} < a\right) \rightarrow P(Z < a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^a e^{-\frac{x^2}{2}} dx$$

as $n \rightarrow \infty$.

Let X_1, X_2, \dots be a sequence of independent random variables having common distribution and let $\mathbb{E}X_i = \mu$ and $\text{Var}(X_i) = \sigma^2$. Then, the distribution of

$$\frac{X_1 + X_2 + \dots + X_n - n\mu}{\sigma\sqrt{n}}$$

tends to the standard normal distribution Z as $n \rightarrow \infty$. That is

$$P\left(\frac{X_1 + X_2 + \dots + X_n - n\mu}{\sigma\sqrt{n}} < a\right) \rightarrow P(Z < a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^a e^{-\frac{x^2}{2}} dx$$

as $n \rightarrow \infty$.

So considering $\bar{X} = \sum X_i/n$ we get that

$$\sqrt{n} \frac{\bar{X} - \mu}{\sigma} \approx Z.$$

So to understand "intervals" for μ we need to understand intervals for Z .

We say that Z is a standard normal random variable if the density function of Z is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Let me remind that in general it is impossible to compute exactly $P(Z \leq z)$ (it is done with help of tables, which we will see on the next slide, which were computed using approximation of integral methods).

We say that Z is a standard normal random variable if the density function of Z is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Let me remind that in general it is impossible to compute exactly $P(Z \leq z)$ (it is done with help of tables, which we will see on the next slide, which were computed using approximation of integral methods). But still there are a few nice observations which may help us in our computations. Note that, making a substitution $y = -x$

$$P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-x^2/2} dx = \frac{1}{\sqrt{2\pi}} \int_{-z}^{\infty} e^{-y^2/2} dy = P(Z \geq -z).$$

We say that Z is a standard normal random variable if the density function of Z is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Let me remind that in general it is impossible to compute exactly $P(Z \leq z)$ (it is done with help of tables, which we will see on the next slide, which were computed using approximation of integral methods). But still there are a few nice observations which may help us in our computations. Note that, making a substitution $y = -x$

$$P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-x^2/2} dx = \frac{1}{\sqrt{2\pi}} \int_{-z}^{\infty} e^{-y^2/2} dy = P(Z \geq -z).$$

Also

$$P(Z \leq z) + P(Z \geq z) = 1.$$

We say that Z is a standard normal random variable if the density function of Z is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Let me remind that in general it is impossible to compute exactly $P(Z \leq z)$ (it is done with help of tables, which we will see on the next slide, which were computed using approximation of integral methods). But still there are a few nice observations which may help us in our computations. Note that, making a substitution $y = -x$

$$P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-x^2/2} dx = \frac{1}{\sqrt{2\pi}} \int_{-z}^{\infty} e^{-y^2/2} dy = P(Z \geq -z).$$

Also

$$P(Z \leq z) + P(Z \geq z) = 1.$$

So, from $P(Z \leq 0) = P(Z \geq 0)$ we get $P(Z \leq 0) = .5$.

We say that Z is a standard normal random variable if the density function of Z is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Let me remind that in general it is impossible to compute exactly $P(Z \leq z)$ (it is done with help of tables, which we will see on the next slide, which were computed using approximation of integral methods). But still there are a few nice observations which may help us in our computations. Note that, making a substitution $y = -x$

$$P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-x^2/2} dx = \frac{1}{\sqrt{2\pi}} \int_{-z}^{\infty} e^{-y^2/2} dy = P(Z \geq -z).$$

Also

$$P(Z \leq z) + P(Z \geq z) = 1.$$

So, from $P(Z \leq 0) = P(Z \geq 0)$ we get $P(Z \leq 0) = .5$.

We also should always remember that

$$P(a < Z < b) = P(Z < b) - P(Z < a).$$

We say that Z is a standard normal random variable if the density function of Z is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Let me remind that in general it is impossible to compute exactly $P(Z \leq z)$ (it is done with help of tables, which we will see on the next slide, which were computed using approximation of integral methods). But still there are a few nice observations which may help us in our computations. Note that, making a substitution $y = -x$

$$P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-x^2/2} dx = \frac{1}{\sqrt{2\pi}} \int_{-z}^{\infty} e^{-y^2/2} dy = P(Z \geq -z).$$

Also

$$P(Z \leq z) + P(Z \geq z) = 1.$$

So, from $P(Z \leq 0) = P(Z \geq 0)$ we get $P(Z \leq 0) = .5$.

We also should always remember that

$$P(a < Z < b) = P(Z < b) - P(Z < a).$$

Moreover,

$$P(-a < Z < a) = P(Z < a) - P(Z < -a) = 1 - P(Z > a) - P(Z < -a) = 1 - 2P(Z < -a).$$

Table entry for z is the area under the standard normal curve to the left of z .

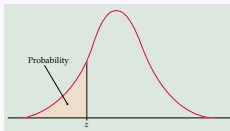


TABLE A Standard normal probabilities

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| -0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

Table- continuation

Table entry for z is the area under the standard normal curve to the left of z .

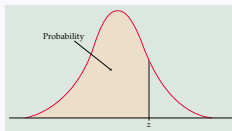


TABLE A Standard normal probabilities (continued)

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

We say that Z is a standard normal random variable if the density function of Z is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Note

$$P(-a < Z < a) = P(Z < a) - P(Z < -a) = 1 - P(Z < -a) - P(Z < -a) = 1 - 2P(Z < -a).$$

We say that Z is a standard normal random variable if the density function of Z is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Note

$$P(-a < Z < a) = P(Z < a) - P(Z < -a) = 1 - P(Z < -a) - P(Z < -a) = 1 - 2P(Z < -a).$$

Find $P(-1 < Z < 1)$.

We say that Z is a standard normal random variable if the density function of Z is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Note

$$P(-a < Z < a) = P(Z < a) - P(Z < -a) = 1 - P(Z < -a) - P(Z < -a) = 1 - 2P(Z < -a).$$

Find $P(-1 < Z < 1)$.

Solution $P(-1 < Z < 1) = 1 - 2P(Z < -1) = 1 - 2 * .1587 = 0.6826.$

Reading Table backwards.

But now we are actually interested to read the table "in opposite order". We are interested to find number a for which

$$P(-a < Z < a) = 1 - \alpha,$$

But now we are actually interested to read the table "in opposite order". We are interested to find number a for which

$$P(-a < Z < a) = 1 - \alpha,$$

i.e. a number a for which random standard normal random variable $Z \in (-a, a)$ with probability $1 - \alpha$.

For this we need to find a :

$$1 - 2P(Z < -a) = 1 - \alpha.$$

But now we are actually interested to read the table "in opposite order". We are interested to find number a for which

$$P(-a < Z < a) = 1 - \alpha,$$

i.e. a number a for which random standard normal random variable $Z \in (-a, a)$ with probability $1 - \alpha$.

For this we need to find a :

$$1 - 2P(Z < -a) = 1 - \alpha.$$

thus

$$P(Z < -a) = \alpha/2.$$

But now we are actually interested to read the table "in opposite order". We are interested to find number a for which

$$P(-a < Z < a) = 1 - \alpha,$$

i.e. a number a for which random standard normal random variable $Z \in (-a, a)$ with probability $1 - \alpha$.

For this we need to find a :

$$1 - 2P(Z < -a) = 1 - \alpha.$$

thus

$$P(Z < -a) = \alpha/2.$$

Such $-a$ is called $-z_{\alpha/2}$ (or just a is called $z_{\alpha/2}$).

But now we are actually interested to read the table "in opposite order". We are interested to find number a for which

$$P(-a < Z < a) = 1 - \alpha,$$

i.e. a number a for which random standard normal random variable $Z \in (-a, a)$ with probability $1 - \alpha$.

For this we need to find a :

$$1 - 2P(Z < -a) = 1 - \alpha.$$

thus

$$P(Z < -a) = \alpha/2.$$

Such $-a$ is called $-z_{\alpha/2}$ (or just a is called $z_{\alpha/2}$).

Find a such that $P(-a < Z < a) = 0.80$.

But now we are actually interested to read the table "in opposite order". We are interested to find number a for which

$$P(-a < Z < a) = 1 - \alpha,$$

i.e. a number a for which random standard normal random variable $Z \in (-a, a)$ with probability $1 - \alpha$.

For this we need to find a :

$$1 - 2P(Z < -a) = 1 - \alpha.$$

thus

$$P(Z < -a) = \alpha/2.$$

Such $-a$ is called $-z_{\alpha/2}$ (or just a is called $z_{\alpha/2}$).

Find a such that $P(-a < Z < a) = 0.80$.

Solution In this case $1 - \alpha = 0.8$ so $\alpha = 0.2$ and $\alpha/2 = 0.1$

But now we are actually interested to read the table "in opposite order". We are interested to find number a for which

$$P(-a < Z < a) = 1 - \alpha,$$

i.e. a number a for which random standard normal random variable $Z \in (-a, a)$ with probability $1 - \alpha$.

For this we need to find a :

$$1 - 2P(Z < -a) = 1 - \alpha.$$

thus

$$P(Z < -a) = \alpha/2.$$

Such $-a$ is called $-z_{\alpha/2}$ (or just a is called $z_{\alpha/2}$).

Find a such that $P(-a < Z < a) = 0.80$.

Solution In this case $1 - \alpha = 0.8$ so $\alpha = 0.2$ and $\alpha/2 = 0.1$ we need to search the table for a value $-z_{0.1}$ i.e. the value for which

$$P(Z < -z_{0.1}) = .1$$

after long search we get $-z_{0.1} \approx -1.28$ and thus $P(-1.28 < Z < 1.28) \approx .80$.

Reading Table backwards: Essential Example.

Find a such that $P(-a < Z < a) = .95$, i.e. a so that we are 95% sure that $Z \in (-a, a)$.

Reading Table backwards: Essential Example.

Find a such that $P(-a < Z < a) = .95$, i.e. a so that we are 95% sure that $Z \in (-a, a)$.

Solution In this case $1 - \alpha = 0.95$ so $\alpha = 0.05$ and $\alpha/2 = 0.025$

Reading Table backwards: Essential Example.

Find a such that $P(-a < Z < a) = .95$, i.e. a so that we are 95% sure that $Z \in (-a, a)$.

Solution In this case $1 - \alpha = 0.95$ so $\alpha = 0.05$ and $\alpha/2 = 0.025$ we need to search the table for a value $-z_{0.025}$ i.e. the value for which

$$P(Z < -z_{0.025}) = 0.025$$

Reading Table backwards: Essential Example.

Find a such that $P(-a < Z < a) = .95$, i.e. a so that we are 95% sure that $Z \in (-a, a)$.

Solution In this case $1 - \alpha = 0.95$ so $\alpha = 0.05$ and $\alpha/2 = 0.025$ we need to search the table for a value $-z_{0.025}$ i.e. the value for which

$$P(Z < -z_{0.025}) = 0.025$$

after (not so) long search we get $-z_{0.025} = -1.96$ and thus

$$P(-1.96 < Z < 1.96) = 0.95.$$

Reading Table backwards: Essential Example.

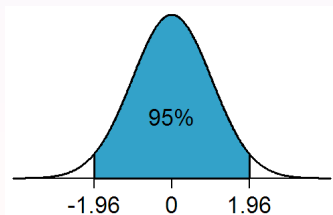
Find a such that $P(-a < Z < a) = .95$, i.e. a so that we are 95% sure that $Z \in (-a, a)$.

Solution In this case $1 - \alpha = 0.95$ so $\alpha = 0.05$ and $\alpha/2 = 0.025$ we need to search the table for a value $-z_{0.025}$ i.e. the value for which

$$P(Z < -z_{0.025}) = 0.025$$

after (not so) long search we get $-z_{0.025} = -1.96$ and thus

$$P(-1.96 < Z < 1.96) = 0.95.$$



Let X_1, \dots, X_n be a sample of size n from a population having known standard deviation σ and suppose that we want to utilize this sample to obtain a 95 percent confidence interval estimator for population mean. We know that (Central Limit Theorem):

Let X_1, \dots, X_n be a sample of size n from a population having known standard deviation σ and suppose that we want to utilize this sample to obtain a 95 percent confidence interval estimator for population mean. We know that (Central Limit Theorem):

$$\sqrt{n} \frac{\bar{X} - \mu}{\sigma} \approx Z.$$

Using $P(|Z| < 1.96) = 0.95$ we can write

Let X_1, \dots, X_n be a sample of size n from a population having known standard deviation σ and suppose that we want to utilize this sample to obtain a 95 percent confidence interval estimator for population mean. We know that (Central Limit Theorem):

$$\sqrt{n} \frac{\bar{X} - \mu}{\sigma} \approx Z.$$

Using $P(|Z| < 1.96) = 0.95$ we can write

$$P\left(\frac{\sqrt{n}}{\sigma} |\bar{X} - \mu| \leq 1.96\right) = 0.95$$

Let X_1, \dots, X_n be a sample of size n from a population having known standard deviation σ and suppose that we want to utilize this sample to obtain a 95 percent confidence interval estimator for population mean. We know that (Central Limit Theorem):

$$\sqrt{n} \frac{\bar{X} - \mu}{\sigma} \approx Z.$$

Using $P(|Z| < 1.96) = 0.95$ we can write

$$P\left(\frac{\sqrt{n}}{\sigma} |\bar{X} - \mu| \leq 1.96\right) = 0.95$$

or

$$P\left(|\bar{X} - \mu| \leq 1.96 \frac{\sigma}{\sqrt{n}}\right) = 0.95.$$

Let X_1, \dots, X_n be a sample of size n from a population having known standard deviation σ and suppose that we want to utilize this sample to obtain a 95 percent confidence interval estimator for population mean. We know that (Central Limit Theorem):

$$\sqrt{n} \frac{\bar{X} - \mu}{\sigma} \approx Z.$$

Using $P(|Z| < 1.96) = 0.95$ we can write

$$P\left(\frac{\sqrt{n}}{\sigma} |\bar{X} - \mu| \leq 1.96\right) = 0.95$$

or

$$P\left(|\bar{X} - \mu| \leq 1.96 \frac{\sigma}{\sqrt{n}}\right) = 0.95.$$

Finally

$$P\left(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}\right) = 0.95.$$

Let X_1, \dots, X_n be a sample of size n from a population having known standard deviation σ and suppose that we want to utilize this sample to obtain a 95 percent confidence interval estimator for population mean. We know that (Central Limit Theorem):

$$\sqrt{n} \frac{\bar{X} - \mu}{\sigma} \approx Z.$$

Using $P(|Z| < 1.96) = 0.95$ we can write

$$P\left(\frac{\sqrt{n}}{\sigma} |\bar{X} - \mu| \leq 1.96\right) = 0.95$$

or

$$P\left(|\bar{X} - \mu| \leq 1.96 \frac{\sigma}{\sqrt{n}}\right) = 0.95.$$

Finally

$$P\left(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}\right) = 0.95.$$

That is, with 95 percent probability the interval $(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}})$ will contain the population mean.

The interval

$$\left(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}} \right)$$

is said to be 95 **percent confidence interval estimator** of the population mean μ .

The interval

$$\left(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

is said to be **95 percent confidence interval estimator** of the population mean μ . If the observed value (i.e. the value we computed) of \bar{X} is \bar{x} then we call the interval

$$\left(\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

a **95 percent confidence interval estimator** of μ .

The interval

$$\left(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

is said to be 95 **percent confidence interval estimator** of the population mean μ . If the observed value (i.e. the value we computed) of \bar{X} is \bar{x} then we call the interval

$$\left(\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

a 95 **percent confidence interval estimator** of μ .

Also traditionally (and to shorten some writing), we denote

$$\left(\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

as **interval** $\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$.

The interval

$$\left(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

is said to be **95 percent confidence interval estimator** of the population mean μ . If the observed value (i.e. the value we computed) of \bar{X} is \bar{x} then we call the interval

$$\left(\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

a **95 percent confidence interval estimator** of μ .

Also traditionally (and to shorten some writing), we denote

$$\left(\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

as **interval** $\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$. This is also made to emphasize that we consider an interval "around" observed value \bar{x} with "radius" $1.96 \frac{\sigma}{\sqrt{n}}$.

Suppose that if a signal having intensity μ originates at location A , then the intensity recorded at location B is normally distributed with mean μ and standard deviation 3. That is, due to "noise", the intensity recorded differs from the actual intensity of the signal by an amount that is normal with mean 0 and standard deviation 3. To reduce the error, the same signal with mean 0 recorded 10 times. If the successive recorded values are

17, 21, 20, 18, 19, 22, 20, 21, 16, 19

construct a 95 percent confidence interval for μ , the actual density.

Suppose that if a signal having intensity μ originates at location A , then the intensity recorded at location B is normally distributed with mean μ and standard deviation 3. That is, due to "noise", the intensity recorded differs from the actual intensity of the signal by an amount that is normal with mean 0 and standard deviation 3. To reduce the error, the same signal with mean 0 recorded 10 times. If the successive recorded values are

17, 21, 20, 18, 19, 22, 20, 21, 16, 19

construct a 95 percent confidence interval for μ , the actual density.

Solution: The value of the sample mean is

$$\bar{x} = \frac{17 + 21 + 20 + 18 + 19 + 22 + 20 + 21 + 16 + 19}{10} = 19.3.$$

Suppose that if a signal having intensity μ originates at location A , then the intensity recorded at location B is normally distributed with mean μ and standard deviation 3. That is, due to "noise", the intensity recorded differs from the actual intensity of the signal by an amount that is normal with mean 0 and standard deviation 3. To reduce the error, the same signal with mean 0 recorded 10 times. If the successive recorded values are

$$17, 21, 20, 18, 19, 22, 20, 21, 16, 19$$

construct a 95 percent confidence interval for μ , the actual density.

Solution: The value of the sample mean is

$$\bar{x} = \frac{17 + 21 + 20 + 18 + 19 + 22 + 20 + 21 + 16 + 19}{10} = 19.3.$$

Since $\sigma = 3$ it follows that 95 percent confidence interval estimate of μ is

$$19.3 \pm 1.96 \frac{3}{\sqrt{10}} = 19.3 \pm 1.86.$$

Suppose that if a signal having intensity μ originates at location A , then the intensity recorded at location B is normally distributed with mean μ and standard deviation 3. That is, due to "noise", the intensity recorded differs from the actual intensity of the signal by an amount that is normal with mean 0 and standard deviation 3. To reduce the error, the same signal with mean 0 recorded 10 times. If the successive recorded values are

$$17, 21, 20, 18, 19, 22, 20, 21, 16, 19$$

construct a 95 percent confidence interval for μ , the actual density.

Solution: The value of the sample mean is

$$\bar{x} = \frac{17 + 21 + 20 + 18 + 19 + 22 + 20 + 21 + 16 + 19}{10} = 19.3.$$

Since $\sigma = 3$ it follows that 95 percent confidence interval estimate of μ is

$$19.3 \pm 1.96 \frac{3}{\sqrt{10}} = 19.3 \pm 1.86.$$

So we can claim with 95 confidence that $\mu \in (17.44, 21.16)$.

We can also consider interval estimators having confidence levels different from 0.95.

We can also consider interval estimators having confidence levels different from 0.95.
Using the definition that

$$P(|Z| \leq z_{\alpha/2}) = P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha.$$

We can also consider interval estimators having confidence levels different from 0.95. Using the definition that

$$P(|Z| \leq z_{\alpha/2}) = P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha.$$

From this we get

$$P\left(\frac{\sqrt{n}}{\sigma} |\bar{X} - \mu| \leq z_{\alpha/2}\right) = 1 - \alpha.$$

We can also consider interval estimators having confidence levels different from 0.95. Using the definition that

$$P(|Z| \leq z_{\alpha/2}) = P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha.$$

From this we get

$$P\left(\frac{\sqrt{n}}{\sigma} |\bar{X} - \mu| \leq z_{\alpha/2}\right) = 1 - \alpha.$$

The same logic as before gives us

The interval $\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ is said to be $100(1 - \alpha)$ **percent confidence interval estimator** of the population mean.

We can also consider interval estimators having confidence levels different from 0.95. Using the definition that

$$P(|Z| \leq z_{\alpha/2}) = P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha.$$

From this we get

$$P\left(\frac{\sqrt{n}}{\sigma} |\bar{X} - \mu| \leq z_{\alpha/2}\right) = 1 - \alpha.$$

The same logic as before gives us

The interval $\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ is said to be $100(1 - \alpha)$ **percent confidence interval estimator** of the population mean.

Confidence Level Percentiles

| Confidence level $100(1 - \alpha)$ | Corresponding α | $z_{\alpha/2}$ |
|------------------------------------|------------------------|---------------------|
| 90 | 0.10 | $z_{0.05} = 1.645$ |
| 95 | 0.05 | $z_{0.025} = 1.960$ |
| 99 | 0.01 | $z_{0.005} = 2.576$ |

Example.

Consider the previous example: to reduce the error, the same signal with mean 0 and standard deviation 3 recorded 10 times

17, 21, 20, 18, 19, 22, 20, 21, 16, 19

Find 90 and 99 percent confidence intervals for μ .

Example.

Consider the previous example: to reduce the error, the same signal with mean 0 and standard deviation 3 recorded 10 times

17, 21, 20, 18, 19, 22, 20, 21, 16, 19

Find 90 and 99 percent confidence intervals for μ .

Solution: We need to construct $100(1 - \alpha)$ confidence intervals with $\alpha = 0.1$ and $\alpha = 0.01$.

Example.

Consider the previous example: to reduce the error, the same signal with mean 0 and standard deviation 3 recorded 10 times

17, 21, 20, 18, 19, 22, 20, 21, 16, 19

Find 90 and 99 percent confidence intervals for μ .

Solution: We need to construct $100(1 - \alpha)$ confidence intervals with $\alpha = 0.1$ and $\alpha = 0.01$. Checking the table (see previous slide) we get

$$z_{0.05} = 1.645 \text{ and } z_{0.005} = 2.576.$$

Example.

Consider the previous example: to reduce the error, the same signal with mean 0 and standard deviation 3 recorded 10 times

17, 21, 20, 18, 19, 22, 20, 21, 16, 19

Find 90 and 99 percent confidence intervals for μ .

Solution: We need to construct $100(1 - \alpha)$ confidence intervals with $\alpha = 0.1$ and $\alpha = 0.01$. Checking the table (see previous slide) we get

$$z_{0.05} = 1.645 \text{ and } z_{0.005} = 2.576.$$

Thus 90 percent confidence interval is

$$\bar{x} \pm 1.645 \frac{\sigma}{\sqrt{n}}$$

Example.

Consider the previous example: to reduce the error, the same signal with mean 0 and standard deviation 3 recorded 10 times

17, 21, 20, 18, 19, 22, 20, 21, 16, 19

Find 90 and 99 percent confidence intervals for μ .

Solution: We need to construct $100(1 - \alpha)$ confidence intervals with $\alpha = 0.1$ and $\alpha = 0.01$. Checking the table (see previous slide) we get

$$z_{0.05} = 1.645 \text{ and } z_{0.005} = 2.576.$$

Thus 90 percent confidence interval is

$$\bar{x} \pm 1.645 \frac{\sigma}{\sqrt{n}}$$

and 99 percent confidence interval is

$$\bar{x} \pm 2.576 \frac{\sigma}{\sqrt{n}}$$

Example.

Consider the previous example: to reduce the error, the same signal with mean 0 and standard deviation 3 recorded 10 times

$$17, 21, 20, 18, 19, 22, 20, 21, 16, 19$$

Find 90 and 99 percent confidence intervals for μ .

Solution: We need to construct $100(1 - \alpha)$ confidence intervals with $\alpha = 0.1$ and $\alpha = 0.01$. Checking the table (see previous slide) we get

$$z_{0.05} = 1.645 \text{ and } z_{0.005} = 2.576.$$

Thus 90 percent confidence interval is

$$\bar{x} \pm 1.645 \frac{\sigma}{\sqrt{n}}$$

and 99 percent confidence interval is

$$\bar{x} \pm 2.576 \frac{\sigma}{\sqrt{n}}$$

Last step is to plug $n = 10$ and $\sigma = 3$ to get that 90 percent confidence interval is

$$19.3 \pm 1.645 \frac{3}{\sqrt{10}} = 19.3 \pm 1.56$$

Example.

Consider the previous example: to reduce the error, the same signal with mean 0 and standard deviation 3 recorded 10 times

$$17, 21, 20, 18, 19, 22, 20, 21, 16, 19$$

Find 90 and 99 percent confidence intervals for μ .

Solution: We need to construct $100(1 - \alpha)$ confidence intervals with $\alpha = 0.1$ and $\alpha = 0.01$. Checking the table (see previous slide) we get

$$z_{0.05} = 1.645 \text{ and } z_{0.005} = 2.576.$$

Thus 90 percent confidence interval is

$$\bar{x} \pm 1.645 \frac{\sigma}{\sqrt{n}}$$

and 99 percent confidence interval is

$$\bar{x} \pm 2.576 \frac{\sigma}{\sqrt{n}}$$

Last step is to plug $n = 10$ and $\sigma = 3$ to get that 90 percent confidence interval is

$$19.3 \pm 1.645 \frac{3}{\sqrt{10}} = 19.3 \pm 1.56$$

and 99 percent confidence interval is

$$19.3 \pm 2.576 \frac{3}{\sqrt{10}} = 19.3 \pm 2.44.$$

Example.

Consider the previous example: to reduce the error, the same signal with mean 0 and standard deviation 3 recorded 10 times

$$17, 21, 20, 18, 19, 22, 20, 21, 16, 19$$

Find 90 and 99 percent confidence intervals for μ .

Solution: We need to construct $100(1 - \alpha)$ confidence intervals with $\alpha = 0.1$ and $\alpha = 0.01$. Checking the table (see previous slide) we get

$$z_{0.05} = 1.645 \text{ and } z_{0.005} = 2.576.$$

Thus 90 percent confidence interval is

$$\bar{x} \pm 1.645 \frac{\sigma}{\sqrt{n}}$$

and 99 percent confidence interval is

$$\bar{x} \pm 2.576 \frac{\sigma}{\sqrt{n}}$$

Last step is to plug $n = 10$ and $\sigma = 3$ to get that 90 percent confidence interval is

$$19.3 \pm 1.645 \frac{3}{\sqrt{10}} = 19.3 \pm 1.56$$

and 99 percent confidence interval is

$$19.3 \pm 2.576 \frac{3}{\sqrt{10}} = 19.3 \pm 2.44.$$

Compare it with 95 percent confidence interval and notice that as we increase the certainty level the interval also increase!