Lecture 9.2, MATH-57091 Probability and Statistics for High-School Teachers.

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$(1-\alpha)$- confidence interval estimator

Using the definition that

$$P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1-\alpha.$$

We get

$$P\left(\frac{\sqrt{n}}{\sigma}|\bar{X}-\mu| \leq z_{\alpha/2}\right) = 1-\alpha.$$

The same logic as before gives us

The interval $\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ is said to be $100(1-\alpha)$ **percent confidence interval estimator** of the population mean.

Confidence Level Percentiles

Confidence level $100(1-\alpha)$	Corresponding α	$z_{\alpha/2}$
90	0.10	$z_{0.05} = 1.645$
95	0.05	$z_{0.025} = 1.960$
99	0.01	$z_{0.005} = 2.576$

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or equivalently

$$n \geq \left(\frac{3.92\sigma}{b}\right)^2.$$

Example:

If the population standard deviation is $\sigma = 2$ and we want a 95 percent confidence interval estimate of the mean μ that is of size less than or equal to $b = 0.1$ how large the sample is needed?

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So we need a sample of size 6147.

General Case:

We can also (using absolutely the same logic) write a more general formula to determine the sample size needed for general $100(1-\alpha)$ confidence intervals:

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We would like to estimate μ, the average nicotine content of newly marketed cigarette. Assume that it is known from the past that the standard deviation from nicotine content is 0.7 milligrams. How large sample is necessary for the length of 99 percent confidence interval to be less that or equal to 0.3 milligrams?

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Solution: We note that $\alpha = 0.01$ and thus

$$n \geq \left(\frac{2z_{.005}\sigma}{b}\right)^2 = \left(\frac{2 * 2.676 * 0.7}{0.3}\right)^2 = 155.9.$$

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So the sample size of at least 156 is needed.

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A $100(1-\alpha)$ percent lower confidence bound for the population mean μ is given by

$$\bar{X} - z_\alpha \frac{\sigma}{\sqrt{n}}.$$

That is, with $100(1-\alpha)$ percent confidence

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We would like to estimate μ, the average nicotine content of newly marketed cigarette. Assume that it is known from the past that the standard deviation from mean nicotine content is 0.7 milligrams. Please, specify a value that, with 95 percent confidence, is less than the average nicotine content, if it is known that the average nicotine finding out of 44 checked cigarets is 1.74 milligrams.

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Solution: The bound we are looking for is given by

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Solution: The bound we are looking for is given by

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Where $\alpha = 0.05$, thus $z_{0.05} = 1.645$ (remember that table in previous lecture 9.1. gives you $-z_{\alpha}$!), $n = 44$, $\sigma = 0.7$ and $\bar{X} = 1.74$

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$$1.74 - 1.645 \frac{0.7}{\sqrt{44}} = 1.57.$$

That is, we can assert, with 95 percent confidence, that the average nicotine content is greater than 1.57 milligrams.

Upper Confidence Bound

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A $100(1-\alpha)$ percent upper confidence bound for the population mean μ is given by

$$\bar{X} + z_\alpha \frac{\sigma}{\sqrt{n}}.$$

That is, with $100(1-\alpha)$ percent confidence

$$\mu < \bar{X} + z_\alpha \frac{\sigma}{\sqrt{n}}.$$

Again, need to estimate μ, the average nicotine content of newly marketed cigarette. Assume that it is known from the past that the standard deviation from the mean nicotine content is 0.7 milligrams. Please, specify a value that, with 95 percent confidence, is greater than the average nicotine content, if it is known that the average nicotine finding out of 44 checked cigarets is 1.74 milligrams.

Solution: The bound in this case is given by

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Solution: The bound in this case is given by

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Where again $\alpha = 0.05$, thus $z_{0.05} = 1.645$, $n = 44$, $\sigma = 0.7$ and $\bar{X} = 1.74$

$$1.74 + 1.645 \frac{0.7}{\sqrt{44}} = 1.91.$$

That is, we can assert, with 95 percent confidence, that the average nicotine content is less than 1.914 milligrams.