Differential Geometry, MATH-45011/55011.
Home Work 2, due on Wednesday, SEPTEMBER 11
Instructor: Prof. Artem Zvavitch

Problem 1. *Compute, curvature, torsion and also tangent, normal and binormal vectors for*

- $\alpha(t) = \left(\frac{1}{3}(1+t)^{3/2}, \frac{1}{3}(1-t)^{3/2}, \frac{t}{\sqrt{2}}\right)$.
- $\alpha(t) = \left(\cos^3 t, \sin^3 t\right)$ *(yes, there is no third coordinate ... or if you wish the third coordinate is always zero).*
- $\alpha(t) = (a\cosh t, a\sinh t, at)$.
- $\frac{x^2}{4} + y^2 9 = 1$.

Problem 2. *Find the curvature of the line given implicitly by the equations*

$$x + \sinh x = siny + y \text{ and } z + e^z = x + \ln(1+x) + 1,$$

at the point $(0,0,0)$.

Problem 3. *One often gives a plane curve in polar coordinates by polar coordinates* $\rho(\theta)$, *where* $\theta \in (a,b)$ *is an angle.*
a) Show that the arclength is

$$\int_a^b \sqrt{\rho^2 + (\rho')^2} d\theta$$

where the prime denotes the derivative relative to θ.
b) Show that the curvature is

$$k(\theta) = \frac{|2(\rho')^2 - \rho\rho'' + \rho^2|}{((\rho')^2 + \rho^2)^{3/2}}.$$