Differential Geometry, MATH-45011/55011.
Home Work 3, due on Wednesday, SEPTEMBER 18
Instructor: Prof. Artem Zvavitch

Problem 1. *Assume that all normals of a regular curve pass through a fixed point. Prove that the curve is a part of the circle.*

Problem 2. *Let $\alpha : I \to \mathbb{R}^3$ be a regular parametrized curve (not necessarily by arc length) and let $\beta : J \to \mathbb{R}^3$ be a re-parametrization of α by the arch length $s = s(t)$. Let $t = t(s)$ be the inverse function of s denote $\frac{d\alpha}{ds} = \alpha'$ and $\frac{d^2\alpha}{ds^2} = \alpha''$.... Prove that*

- $\frac{dt}{ds} = \frac{1}{|\alpha'|}$ *and* $\frac{d^2t}{ds^2} = -\frac{\alpha'\cdot\alpha''}{|\alpha'|^4}$.
- $k(t) = \frac{|\alpha'\wedge\alpha''|}{|\alpha'|^3}$
- $\tau(t) = -\frac{(\alpha'\wedge\alpha'')\cdot\alpha'''}{|\alpha'|^2}$

Problem 3. *(*) Assume that $\tau(s) \neq 0$ and $k'(s) \neq 0$ for all $s \in I$. Show that a necessary and sufficient condition for curve α to lie on a sphere is that*

$$\frac{1}{k^2} + \left(\frac{\frac{d}{ds}\left(\frac{1}{k}\right)}{\tau}\right)^2 = const.$$