

**Differential Geometry, MATH-45011/55011.**  
**Home Work 5, due on Wednesday, October 9**  
**OUT OF 130 points**  
**Instructor: Prof. Artem Zvavitch**

**Problem 1.** Calculate the Frenet apparatus  $(\kappa, \tau, T, N, B)$  for the curve

$$\alpha(t) = (e^t \cos t, e^t \sin t, e^t).$$

Also, please, identify if this curve planar or not.

**Problem 2.** Consider the graph  $y = f(x)$ ,  $x \in [a, b]$ , where  $f$  is a smooth function. Please, find formulas for the Frenet apparatus  $(\kappa, \tau, T, N, B)$  for the curve. Please, also, prove that the arch length is given by the traditional formula:

$$\text{length} = \int_a^b \sqrt{1 + (f'(x))^2} dx.$$

**Problem 3.** A regular parametrized curve  $\alpha$  has the property that all its tangent lines pass through a fixed point. Prove that  $\alpha$  is a part of the straight line. What do you think would happen if we remove the regularity assumption?

**Problem 4.** Show that the cylinder  $\{(x, y, z) \in \mathbb{R}^3 : \frac{x^2}{9} + \frac{y^2}{16} = 1\}$  is a regular surface.