

Differential Geometry, MATH-45011/55011.
Home Work 6, due on Wednesday, October 16
Instructor: Prof. Artem Zvavitch

Problem 1. *Compute the first fundamental forms of the following parametrized surfaces, when they are regular*

- $x(u, v) = (a \sin u \cos v, b \sin u \sin v, c \cos u)$, $a, b, c > 0$ - ellipsoid.
- $x(u, v) = (au \cos v, bu \sin v, u^2)$ - elliptic paraboloid.
- Torus.

Problem 2. *Given the parametrized surface*

$$x(u, v) = (u \cos v, u \sin v, \log \cos v + u), \text{ for } v \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right),$$

show that the two curves $x(u_1, v), x(u_2, v)$ determine segments of equal lengths on all curves $x(u, \text{const})$.

Problem 3. *Show that the area A of bounded region R of the surface $z = f(x, y)$ is*

$$A = \int \int_Q \sqrt{1 + f_x^2 + f_y^2} dx dy,$$

where $f(Q) = R$.