Differential Geometry, MATH-45011/55011.
Home Work 7, due on Wednesday, October 23
Instructor: Prof. Artem Zvavitch

Problem 1. *Let* $P = \{(x, y, z) \in \mathbb{R}^3; z = 0\}$ *be the* xy*-plane and let* $\mathbf{x} : U \to P$ *be a parametrization*

$$\mathbf{x}(\rho, \theta) = (\rho \cos\theta, \rho \sin\theta),$$

where

$$U = \{(\rho, \theta) \in \mathbb{R}^2; \rho > 0, \theta \in (0, 2\pi)\}.$$

Compute the coefficients of the first fundamental form of P *in this parametrization.*

Problem 2. (Grayson's question) *Let* S *be a surface of revolution and* C *its generating curve. Let* s *be the arc length of* C *and denote by* $\rho = \rho(s)$ *the distance to the rotation axis of the point of* C *corresponding to* s.

- **(Pappus' Theorem.)** *Show that the area of* S *is*

  $$2\pi \int_0^l \rho(s) ds,$$

  where l *is the length of* C.
- *Apply the above formula to compute the area of torus of revolution.*

Problem 3. (Grayson's question, version II) *Let* $\alpha : I \to \mathbb{R}^3$ *be a regular parametrized curve with nonzero curvature everywhere and arc length as parameter. Let*

$$x(s, v) = \alpha(s) + r(N(s) \cos v + N(s) \sin v), \qquad r = const. \neq 0, s \in I,$$

be a parametrized surface (the **tube** *of radius* r *around* α*), where* N *is the normal vector and* B *is the binormal vector of* α.

- *Show that, when* x *is regular, its unit normal vector is*

  $$N(s, v) = -(N(s) \cos v + B(s) \sin v).$$
- *Show that the area of the tube is* $2\pi r$ *times the length of the curve* α.