

Exam 1 Maximal Grade is 100pts. You may use Notes, but must provide ALL details. If you use theorem/formula from the notes/book, please, provide EXACT statement. Due in class on Monday Feb 25 OR ANY time before class in Artem's mailbox OR email

Problem 1. Let X and Y be two independent identically distributed random variables. Assume

$$P(X = 0) = 1/3; P(X = 1) = 1/4; P(X = 2) = 1/4; P(X = 3) = 1/6.$$

Please find

- (5 pts) $\mathbb{P}(X \geq 2)$.
- (10pts) $\mathbb{P}(X = Y)$.
- (10pts) $\mathbb{P}(X < Y)$.

Problem 2. Consider independent identically distributed exponential random variable X and Y with rate λ , please find:

- (5pts) $\mathbb{P}(X < -1)$.
- (5pts) $\mathbb{P}(X < 2)$.
- (5pts) $\mathbb{P}(X > 2)$.
- (10pts) Find density function $f_{(X|X+Y)}(x|t)$.

Problem 3. (20pts) Consider two independent uniformly distributed in interval $[0, 1]$ random variables X and Y . Find

$$P(\max(X, Y) < y | \min(X, Y) < x)$$

Problem 4. Artem loves coffee. Every day he drinks 3 or 5 cups of coffee. The number of cups Artem drinks on any given day depends only on the number of cups Artem drunk on the previous day. If Today he had 3 cups then the probability that he will have 3 cups tomorrow is $\frac{1}{3}$. If Today he had 5 cups then the probability that he will have 5 cups tomorrow is $\frac{1}{2}$. Please

- (5pts) Model Artem's coffee addiction as a Markov chain.
- (5pts) Specify classes of this Markov chain.
- (5pts) Assume on Tuesday Artem had 3 cups of coffee. What is the probability that he will have 5 cups of coffee in two days on Thursday?
- (5pts) In the long run how often Artem have 5 cups of coffee?
- (5pts) and, in average, how many cups of coffee per day Artem drinks?

Problem 5. (10pts) Three out of every four trucks on the road are followed by a car, while only one out of every five cars is followed by a truck. What fraction of vehicles on the road are trucks?