<u>Exam 2</u> **8 problems. Each problem 14pts. Maximal Grade is 100pts. You may use Notes, but must provide ALL details. If you use theorem/formula from the notes/book, please, provide EXACT statement. Due in class on Wednesday, April 4 OR ANY time before class in Artem's mailbox OR email**

Problem 1. *Consider exponential random variable X with rate λ and independent exponential random variable Y with rate μ, please find:*

- *Moment generating function, i.e.* $\phi(t) = \mathbb{E}e^{tX}$,
- $\mathbb{E}(X+Y)^2$
- $Var(X)$,
- $\mathbb{E}[X|X<1]$
- $\mathbb{P}(X<Y)$,

Problem 2. *Consider the Markov chain with states $\{0,1,2,3,4\}$ and transitional probabilities*

$$\mathbf{P} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}.$$

Specify the classes of this above Markov chain and determine whether they are transient or recurrent. Find $\mathbf{P_T}$ (transition probabilities from transient state to transient state). Find an expected number of times the chain will be in state 4 given that it starts at state 3? What is the probability that the chain ever comes to state 3 starting at state 4? What is the probability that the chain ever comes to state 3 starting at state 1?

Problem 3. *Consider the Markov chain with states $\{1,2,3\}$ and transitional probabilities*

$$\mathbf{P} = \begin{pmatrix} .4 & .5 & .1 \\ .3 & .2 & .5 \\ .1 & .6 & .3 \end{pmatrix}$$

Find $\mathbb{P}(X_6 = 2|X_4 = 3)$, also in the long run, what proportion of time is the process in each of the three states?

Problem 4. *Please find π_0 for the branching processes when* $P_0 = 1/12, P_1 = 1/4, P_2 = 2/3$.

Problem 5. *Assume that finally we will find that there is some life on the Moon. Assume that habitants of the Moon reproduce with respect to the branching processes with parameters $P_0 = .1, P_1 = .4, P_2 = .5$, find the probability that population of the Moon will (eventually) distinct.*

Problem 6. *Let $\{N(t), t \geq 0\}$ be a Poisson process with rate μ. Let S_n denote the time of the nth event. Find*

- $\mathbb{E}S_3$,
- $\mathbb{E}[S_3|N(1) = 2]$,
- $\mathbb{E}[N(3) - N(1)|N(1) = 3]$.

Problem 7. *Consider a birth and death process with birth rates* $\lambda_i = (i+2)\lambda$, $i \geq 0$, *and death rates* $\mu_i = (i+1)\mu$, $i > 0$, *where* $\lambda, \mu > 0$. *Determine the expected time to go from state* 0 *to state* 4.

Problem 8. *Assume that little Tom gets presents from his mother and father with accordance to two independent poisson processes each having rate* 1 *present in thirty days. Please*

- *Tom is very worried to find out when will he get the next present. Find the probability that he will get his next present in less then 10 days.*
- *Find the probability that Johny would get more then three presents in January (there are 31 days in January).*
- *Find the probability that Tom would get a present from mother before getting a present from father.*