

Analysis II (42002/52002)
Exam 1/ PREPARATION
Instructor: Prof. Artem Zvavitch
due Wednesday, February 15.

**There are 6 problems, each problem 4 points. ALL OF THIS
IS YOUR BONUS!**

Problem 1. *Prove that for any $x, y \in \mathbb{R}$*

$$|\cos 2x - \cos 2y| \leq 2|x - y|.$$

Problem 2. *Find the limit*

$$\lim_{x \rightarrow 0^+} (\sin x)^{\sin x}.$$

Problem 3. *Let $f(x) = |x|^3$, show that f is twice differentiable at 0. Find $f'(0)$ and $f''(0)$, next prove that f is a convex function on \mathbb{R} .*

Problem 4. *A function f is called odd if $f(x) = -f(-x)$ for all $x \in \mathbb{R}$. Assume f is odd and twice differentiable on \mathbb{R} . Please, find $f''(0)$. Next assume that, in addition, f is also a convex function, find $f''(7)$.*

Problem 5. *Consider function $f(x) = 10x^3 + 80x - 7$, show that f^{-1} (the inverse function of f) exists, monotone increasing and differentiable for all $x \in \mathbb{R}$. Find $(f^{-1})'(-7)$.*

Problem 6. *Consider function f differentiable on $[0, \infty]$ such that $f(0) = 0$ and $f'(x) \geq 1$ for all $x \geq 0$. Show that $f(x) \geq x$.*