Theory of Matrices 5/41021
Exam 1/ PREPARATION
Instructor: Prof. Artem Zvavitch
Thursday, February 16.
There are 6 problems, each problem 4 points. ALL OF THIS is YOUR BONUS!

Problem 1. *Consider matrix matrix* $A = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 1 & 2 & 5 & 2 \\ 2 & 2 & 6 & 1 \\ 1 & 3 & 7 & 4 \end{bmatrix}$

(1) *Find a basis for the row space of* A.
(2) *Find the rank of* A.
(3) *Find a basis for the column space of* A.

Problem 2. *Check if* $U = \{(a, b, c) \in \mathbb{R}^3 \mid ab = 0\}$ *is a subspace of* $\mathbb{R}^3$.

Problem 3. *Prove that* $V = \{(a, b, c) \in \mathbb{R}^3 \mid 2a - b + 3c = 0\}$ *is a subspace of* $\mathbb{R}^3$ *find a basis and the dimension of* V.

Problem 4. *Let* u *and* w *be vectors in a vector space* V. *Given that* $S = \{u, w\}$ *is linearly independent, show that* $T = \{u, u+w\}$ *is linearly independent as well. Use definitions only. Write your argument in complete sentences.*

Problem 5. *Let* $S = \{u_1, u_2, u_3, u_4\}$ *be a subset of a vector space* V, *where* $u_2 = 0$. *Prove that the set* S *is linearly dependent. Use definitions only. Write your argument in complete sentences.*

WAIT TILL TUESDAY TO DO THIS

Problem 6. *Let* U *and* W *be subspaces of* $M_2(\mathbb{R})$ *given by*

$$U = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \mid a, b \in \mathbb{R} \right\}, \quad W = \left\{ \begin{bmatrix} 0 & 0 \\ c & d \end{bmatrix} \mid c, d \in \mathbb{R} \right\}.$$

Prove that $M_2(\mathbb{R}) = U \oplus W$.