

Theory of Matrices 5/41021
Exam 2/ PREPARATION
Instructor: Prof. Artem Zvavitch
Thursday, March 23.

There are 8 problems, each problem 4 points. ALL OF THIS is YOUR BONUS! Note actual exam will have LESS problems

Problem 1. Let W be a subspace of \mathbb{R}^4 spanned by the vectors $(1, -1, 0, 1)$ and $(2, 1, -1, 1)$.

- (a) Find a basis of the orthogonal complement of W .
- (b) Orthogonalize the basis you found in (a) using Gram-Schmidt orthogonalization process.

Problem 2. Consider \mathbb{R}^3 as a real inner product space with the inner product being the dot-product.

- (a) Check that the set $B = \{(1, -1, 2), (0, 2, 1), (5, 1, -2)\}$ is orthogonal. Use this to explain (without any additional computations) why B is a basis.
- (b) Use (a) to express $v = (1, 1, 1)$ as a linear combination of vectors in B .

Problem 3. Let V be the vector space $P(t)$ of all polynomials in t over \mathbb{R} , with inner product defined by $\langle f(t), g(t) \rangle = \int_0^1 f(t)g(t)dt$. Let W be the subspace $P_2(t)$ of V .

- (a) Show that $B = \{1, 2t - 1, 6t^2 - 6t + 1\}$ is an orthogonal basis for W .
- (b) Find the projection of $f(t) = t^3$ onto W .

Problem 4. Let U be the subspace of \mathbb{R}^4 spanned by the vectors $u_1 = (1, 2, 0, 1)$, $u_2 = (1, 1, 1, -1)$, and $u_3 = (1, 2, 1, 2)$.

- (a) Apply the Gram-Schmidt algorithm to find an orthonormal basis for U .
- (b) Find the projection of the vector $v = (1, 2, 3, 4)$ onto U .

Problem 5. Find all values of k such that the matrix

$$\begin{bmatrix} 4 & -1 \\ -1 & k \end{bmatrix}$$

is positive definite.

Problem 6. Given that both sets $\{u, v\}$ and $\{u + 2v, u - 2v\}$ are orthogonal, show that u is twice as long as v . That is, show that $\|u\| = 2\|v\|$.

Problem 7. Let $\varphi: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear map such that $(1, 1)$ and $(1, -1)$ are both in the kernel of φ . What can you say about $\varphi(2, 0)$? What can you say about φ ?

Problem 8. Let U and V be vector spaces over \mathbb{R} and let $\varphi: U \rightarrow V$ be a linear map. Show that $\text{Im}(\varphi)$ is a subspace of V .