

Graph Theory and Combinatorics MATH-42021/52021.

Home Work 3, due on Tuesday, July 19

Instructor: Prof. Artem Zvavitch

13 points (yes, 3 extra points!)

Problem 1. Build 6-vertex graphs with the following degrees of vertices, if possible. If not possible, explain why not:

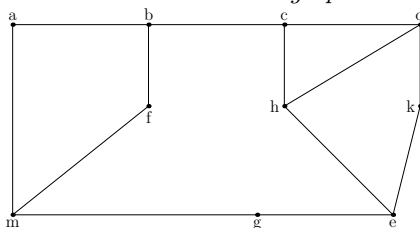
- 5, 5, 5, 5, 5, 5.
- 4, 4, 4, 4, 4, 4.
- 2, 2, 2, 2, 2, 2.
- 3, 3, 3, 1, 1, 1.
- 1, 2, 2, 3, 4, 5.
- 2, 2, 4, 4, 4, 4.

Can you make your examples connected? Planar?

Problem 2. Prove that every connected planar graph with less than 12 vertices has a vertex of degree at most 4.

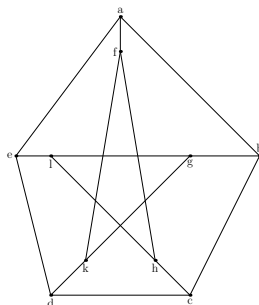
Problem 3. Prove that if a connected graph has $2k$ vertices of odd degree, then there are k disjoint trails that contain all edges.

Problem 4. Consider the graph G



- Does this graph contain Euler cycle?
- Does this graph contain Euler trail?

Problem 5. Please, TRY to remove some edges from the graph below, in a such way that a new graph would contain and Euler cycle (note that the graph must stay connected). Explain, if this is impossible (**this part is Tricky!**).



Problem 6. In chess a "knight move" consists of two squares either vertically or horizontally and then one square in a perpendicular direction. Depending on where the knight is situated, he has a minimum mobility of two moves - when in a corner- and a maximum mobility of eight moves when near the center. Let C be a graph

with 64 vertices corresponding to the squares of a chessboard. Let two vertices of C be joint by an edge whenever a knight can go from one of the corresponding squares to the other in the move. Does C have an Euler trail? (You don't have to draw C to answer!!!!)