

Graph Theory and Combinatorics MATH-42021/52021.

Home Work 5, due on Tuesday, July 26

Instructor: Prof. Artem Zvavitch

10 points (+ an extra problem for 5 points)

Problem 1. Consider a graph G with n vertices:

- If q is the size of the largest independent set in graph G , show that $q\chi(G) \geq n$.
- Use the previous result to prove that $\chi(G)(n - d) \geq n$ and thuse $\chi(G) \geq n/(n - d)$, where d is the minimal degree of the vertex in G .
- Prove that $\chi(G) + \chi(\bar{G}) \leq n + 1$ (**Hint: Induction**).
- $\chi(G)\chi(\bar{G}) \geq n$.
- Use previous inequality to show that $\chi(G) + \chi(\bar{G}) \geq 2\sqrt{n}$.

Problem 2. Extra Show that there exists a planar graph whose every vertex has degree 5 or more.