

Graph Theory and Combinatorics MATH-42021/52021.

Home Work 5, due on Tuesday, July 6

Instructor: Prof. Artem Zvavitch

10 points (+ an extra problem for 5 points)

**Problem 1.** Consider a graph  $G$  with  $n$  vertices:

- If  $q$  is the size of the largest independent set in graph  $G$ , show that  $q\chi(G) \geq n$ .
- Use the previous result to prove that  $\chi(G)(n - d) \geq n$  and thuse  $\chi(G) \geq n/(n - d)$ , where  $d$  is the minimal degree of the vertex in  $G$ .
- Prove that  $\chi(G) + \chi(\bar{G}) \leq n + 1$  (**Hint: Induction**).
- $\chi(G)\chi(\bar{G}) \geq n$ .
- Use previous inequality to show that  $\chi(G) + \chi(\bar{G}) \geq 2\sqrt{n}$ .

**Problem 2. (Extra)** Show that there exists a planar graph whose every vertex has degree 5 or more.