

Graph Theory and Combinatorics MATH-42021/52021.

Home Work 9, due on Monday, December 2

OUT OF 30 points

Instructor: Prof. Artem Zvavitch

Problem 1. Find the coefficient of x^{25} in $(1 + x + x^8)^{10}$

Problem 2. Find the coefficient of x^{12} in

$$(1 - x^2)^{-5},$$

what can you set about the coefficient of x^{17} .

Problem 3. Build a generating function for a_r , the number of r selections from a pile of three red, five black and four white balls.

Problem 4. How many ways are there to get a sum of 25 when 10 distinct dice are rolled?

Problem 5. Use the generating function to show that

$$\sum_{k=0}^m \binom{m}{k} \binom{n}{r-k} = \binom{m+n}{r}.$$

Problem 6. Find functional equations for the generating functions whose coefficients satisfy the relation:

$$a_0 = a_1 = 1 \text{ and } a_n = 2a_{n-1} - 2a_{n-2} + 2 \text{ for } n = 2, 3, \dots$$