

Graph Theory and Combinatorics MATH-42021/52021.

Home Work 5, due on Wednesday, October 9

Instructor: Prof. Artem Zvavitch

Problem 1. *Does there exist a nonplanar graph with chromatic number one? two? three? four?*

Problem 2. *Consider a graph G with n vertices:*

- *If q is the size of the largest independent set in graph G , show that $q\chi(G) \geq n$.*
- *Use the previous result to prove that $\chi(G)(n-d) \geq n$ and thus $\chi(G) \geq n/(n-d)$, where d is the minimal degree of the vertex in G .*
- *Prove that $\chi(G) + \chi(\bar{G}) \leq n + 1$ (Hint: Induction).*
- *$\chi(G)\chi(\bar{G}) \geq n$.*
- *Use previous inequality to show that $\chi(G) + \chi(\bar{G}) \geq 2\sqrt{n}$.*

Problem 3. Extra! do it to get 10 points to exam one (maximal grade will still be 100!) *Show that there exists a planar graph whose every vertex has degree 5 or more.*