

**Graph Theory and Combinatorics MATH-42021/52021.**

**Home Work 6, due on Wednesday, October 16**

**Instructor: Prof. Artem Zvavitch**

**Problem 1.** *Draw all nonisomorphic trees with six vertices.*

**Problem 2.** *Consider an undirected connected graph  $G$  such that the number of edges in  $G$  is less than the number of vertices, show that  $G$  is a tree.*

**Problem 3.** *What is the maximum number of vertices (internal and leaves) in an  $m$ -ary tree of height  $h$ ?*

**Problem 4.** *A forest is an unconnected graph that is a disjoint union of trees. If  $G$  is an  $n$ -vertex forest of  $t$  trees, how many edges does it have?*

**Problem 5.** *Suppose that a chain letter is started by someone in the first week of the year. Each recipient of the chain letter mails copies on to five other people in the next week. After six weeks, how much money in postage (say 44 cents per letter) has been spent on these chain letters?*

**Problem 6. (additional problems +10 pts for exam IF you need it)** *A tree whose edge set is a subset of the edge set of the graph  $G$  is called a spanning tree of  $G$  if the tree has exactly the same vertex set as  $G$ . Does every connected graph have a spanning tree? Either give a proof or a counter-example.*