Graph Theory and Combinatorics MATH-42021/52021.
Home Work 7, due on Wednesday, October 23
Instructor: Prof. Artem Zvavitch

Problem 1. *Show that the sum of the level numbers of all l leaves in a binary tree is at least $l\lceil \log_2 l \rceil$, and hence the average leaf level is at least $\lceil \log_2 l \rceil$.*

Problem 2. *Let T be an undirected tree. If the choice of vertex x to be the root yields a rooted tree of minimal height, then x is called a* **center** *of T. Show that any undirected tree has at most two centers.*

Problem 3. *Please, recover the tree from Prufer sequence $\{3, 4, 5, 5, 3, 6, 7\}$.*

Problem 4. *How many different sequences of heads and tails are possible if a coin is fllipped n times?*

Problem 5. *How many ways there to pick a man and a woman who are not husband and wife from a group of n married couples?*

Problem 6. *How many different six digit numbers can be formed by various arrangements of the six digits $1, 1, 1, 2, 2, 0$ (note: a number can not start with zero).*

Problem 7. *How many n-digit binary sequences are there without any pair of consecutive digits being the same?*

Problem 8. (+10pts for exam, if you need it) *What is the probability that 2 (or more) people in a random group of 25 people have a common birthday? Can you provide a general formula? I.e. What is the probability that 2 (or more) people in a random group of n people have a common birthday? What happens with the answer when n is huge?*