

Graph Theory and Combinatorics MATH-42021/52021.

Home Work 9, due on Monday, November 18

OUT OF 50 points, i.e. there are extra 20 points

Instructor: Prof. Artem Zvavitch

Problem 1. *How many integer solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 15$, such that each $x_i \geq -2$.*

Problem 2. *How many integer non-negative solutions are there to $2x_1 + 3x_2 + x_3 + x_4 + x_5 = 15$.*

Problem 3. *How many integer non-negative solutions are there to the pair of equations $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 28$ and $x_1 + x_2 + x_3 = 9$.*

Problem 4. *How many ways are there to distribute 15 distinct oranges into three different boxes with at most 8 oranges in a box?*

Problem 5. *How many binary sequences of length n are there which contain exactly m occurrences of pattern 01.*

Problem 6. *Prove that*

$$\binom{2n}{n} = 2\binom{n}{2} + n^2.$$

Problem 7. *Prove that*

$$\binom{n}{1} + 6\binom{n}{2} + 6\binom{n}{3} = n^3.$$

Problem 8. *Evaluate*

$$1^3 + 2^3 + 3^3 + \dots + n^3.$$

Problem 9. *Evaluate*

$$\sum_{k=1}^n k3^k \binom{n}{k}$$