

Introduction to Analysis 2
Home Work 2, due Friday, February 3.
Instructor: Prof. Artem Zvavitch

Problem 1. *Prove that for any real numbers x, y :*

$$|\sin x - \sin y| \leq |x - y|$$

also prove that for any $x, y \geq 0$

$$|e^x - e^y| \geq |x - y|.$$

Problem 2. *Find an example of a function which is differentiable on $[-1, 1]$, but not twice differentiable on $[-1, 1]$.*

Problem 3. *Prove that the derivative of the function*

$$f(x) = e^x(x - 1)(x - 2)(x - 3)(x - 4)(x - 5)$$

has at least four roots.

Problem 4. *Find*

- $\lim_{x \rightarrow 0} \frac{\sin x^2}{x}$
- $\lim_{x \rightarrow \infty} \frac{\log x}{e^x}$
- $\lim_{x \rightarrow 0} \frac{\sin x + 1}{x - 2}$

Problem 5. *Show that the sum of two convex functions is again a convex function. Is the same statement true for product? Construct a convex function defined on \mathbb{R} which is increasing for all $x \in \mathbb{R}$.*