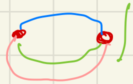
Hint for problem 3

from Homework 3

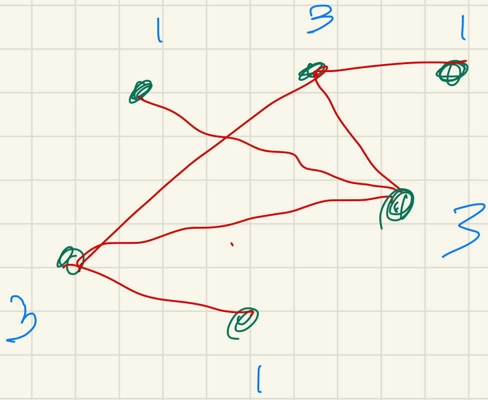
Hint for problem 3 from H.W. 3

The idea is to use the theorem about Euler cycle from Lecture 6. This theorem tell us that an undirected multigraph (i.e may have multiple edges between two vertices ) which is connected may have an Euler cycle (cycle that contains all edges in a graph - do not forget that in a cycle we may use every edge just once, see Lecture 6) if and only if all vertices are of even degree. What it has to do with our problem? We have 2k (even number) of vertices each of odd degree

Example: $\underline{G}$

$6 = 2 \cdot \underset{k}{3}$ vertices

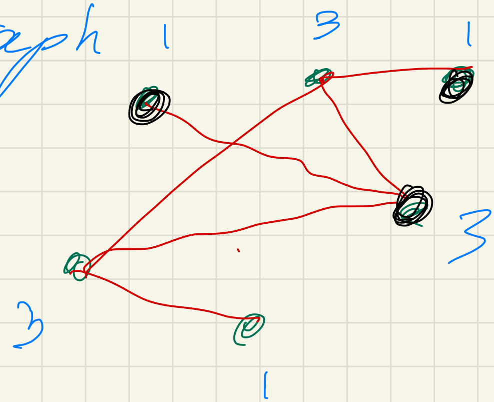
degrees in red



Because G has $2k$ vertices we can split vertices in two disjoint sets A and B each set containing exactly k vertices (it does <u>Not</u> matter how) for example above graph

● – A

● – B



more generally if vertices are $v_1, v_2, v_3, \dots, v_{2k}$ put vertex with even index to A with odd to B. Now

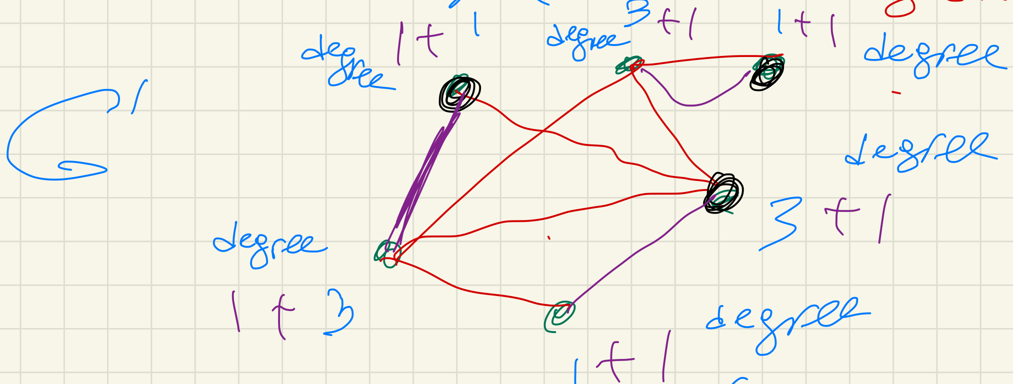
Create a new graph G' connected exactly one vertex from A with a vertex from B by an additional edge, for example, add edges (v_1, v_2)

(v_3, v_4)

$\vdots$

(v_{2k-1}, v_k)

or in our example (New edges in ●



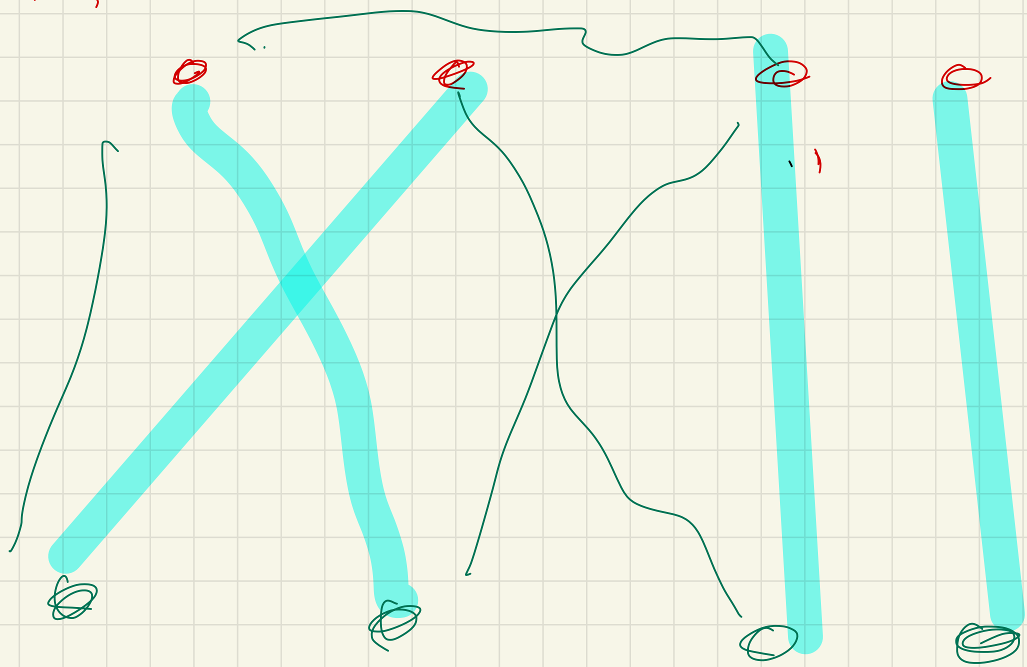
Note that the degree of each vertex will increase exactly by 1. Now G' has vertices of even degree only. Thus, must contain an Euler cycle, call it U

i.e the cycle U visits every vertex once and uses all edges. Now you must go back to graph G for this you should remove all new edges (the edges (V_1, V_2); (V_3, V_4) ... (V_{2k-1}, V_{2k}) you have added when constructing G'. Please think what will happen to U when you remove k edges!

You will get k separated trails that use all vertices

K

Group 1



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Group 2

add one: odd ~> even