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Lecture 1

$x \in A$ — belongs $\qquad x \notin A$ — does not belongs

$1 \in \{1,2,3\}$ $\qquad 6 \notin \{1,2,3\}$

$\forall$ - every $\qquad \Rightarrow$ ← follows

$\exists$ exists

If for every $x \in A \Rightarrow x \in B$

then $A \subseteq B$ — subset of B $\qquad$ (If $\forall x : x \in A \Rightarrow x \in B$ then $A \subseteq B$)

$A \subset B$ — Proper subset: $A \subseteq B$ and $\exists x \in B : x \notin A$

Ex $\{1,2\} \subset \{1,2,3\}$

Def: $A \overset{\text{equal}}{=} B$ if $A \subseteq B$ & $B \subseteq A$.

Some times we defined set using propery (P):

$$A = \{x : P(x)\}$$

example $A = \{x : x^2 > 7\}$

$$A = \{x : x^3 + 6x - 7 = 0\}$$

Natural numbers $\mathbb{N} := \{1, 2, 3, \dots\}$

Integer $\mathbb{Z} = \{0, 1, -1, -2, 2, \dots\}$

Rational $\mathbb{Q} = \{\frac{m}{n} : m, n \in \mathbb{Z}, n \neq 0\}$

Real numbers $\mathbb{R}$ - (to be defined)

Ex of sets

$$A = \{ n \in \mathbb{N} : n < 6 \}$$
$$= \{1, 2, 3, 4, 5\}$$

$$B = \{ 2k : k \in \mathbb{N} \}$$

Set operations

$$A \cup B = \{ x : x \in A \text{ or } x \in B \}$$

$$A \cup B = \{ x : x \in A \text{ and } x \in B \}$$

Complement → Relative to A or A "minus" B

$$A \setminus B = \{ x : x \in A \text{ and } x \notin B \}$$

Emty set " $\emptyset$ " set that have no ellements.

If $A \cap B = \emptyset \Leftarrow A, B$ disjoint.

De Morgan laws:

a) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

b) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

Pf let $x \in A \setminus (B \cup C)$

then $x \in A$ and $x \notin B \cup C$

then $x \in A$ and $x \notin B$ and $x \notin C$

$\Rightarrow (x \in A$ and $x \notin B)$ And $(x \in A$ and $x \notin C)$

$\Rightarrow x \in (A \setminus B) \cap (A \setminus C)$

conversly $y \in (A \setminus B) \cap (A \setminus C)$

$\Rightarrow y \in (A \setminus B)$ and $y \in (A \setminus C)$

$\Rightarrow (y \in A$ and $y \notin B)$ and $(y \in A$ and $y \notin C)$

$\Rightarrow y \in A$ and $y \notin B \cup C$.

(b) – H.W

$$A \cup B \cup C = (A \cup B) \cup C$$

$$A \cap B \cap C = (A \cap B) \cap C$$

We can also write

$$\bigcup_{n=1}^{\infty} A_n := \{x : x \in A_n \text{ for some } n \in \mathbb{N}\}$$

or $\bigcap_{i=1}^{\infty} A_n = \{ x : x \in A_n \text{ for all } n \in N \}$

<u>Ex</u>

$$A_n = \{ x : x \in \mathbb{N} , \; x > n \}$$

$$\bigcup_{n=1}^{\infty} A_n = \mathbb{N} - \{1\}$$

$$\bigcap_{n=1}^{\infty} A_n = \emptyset .$$

Cartesian Product:

$A \times B$ = set or ORdered pairs (a, b).

$= \{ (a, b) , \; a \in A, \; b \in B \}$

ex $\{1, 2, 3\} \times \{ a, b \}$

ex $\{ x : 1 \le x \le 2 \} \times \{ \cancel{1} \le \cancel{x} \le \cancel{0} \}$

Function or mapping.

$f : A \to B$ a rule of correspondance

that assign each $a \in A$ unique $f(a) = b \in B$

<u>Ex</u> $f(x) = x^2$, $A = [-1, 1]$
$B = [0, 10]$

<u>More precise:</u>

f - the set of ordered pairs

$(a, b) \in f \subseteq A \times B$

so that for each $a \in A$ there exist unique $b \in B$.

f - mapping

Image $f : A \to B$

$E \subseteq A$

$f(E) = \{f(a) : a \in E\}$

Inverse image : if $H \subseteq B$

$f^{-1}(H) = \{a : f(a) \in H\}$

Ex $f : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = x^2$.

$E = \{x : 0 \le x \le 2\}$

$f(E) = \{f(x),\ x \in E\}$

$G = \{x : 0 \le x \le 4\}$

$f^{-1}(G) = \{y : \exists x \in G\ f(x) = y\}$

Ex $f : A \to B$, $\quad G, H \subseteq B$

$$f^{-1}(G \cap H) \subseteq f^{-1}(G) \cap f^{-1}(H)$$

(opposite is also true H.W)

~~Proof~~ **Pf** if $y \in f^{-1}(G \cap H)$

$\overset{def}{\Longrightarrow} \exists x \in G \cap H : f(x) = y$

$\Rightarrow \exists x \in G \quad f(x) = y$
$\quad\ \exists x \in H \quad f(x) = y$

$$\Rightarrow \left.\begin{array}{l} y \in f^{-1}(G) \\ y \in f^{-1}(H) \end{array}\right\} \Rightarrow y \in f^{-1}(G) \cap f^{-1}(H)$$