Lecture 2

Function A, B-sets. The function from A to B is a set f of ordered pairs in $A \times B$ such that for each $a \in A$
$\exists!$ $b \in B$: $(a, b) \in f$.
φ
(exists unique)

Ex $A = (1, 2, 3)$
$B = (c, d)$

$$A \times B = \{(1, c), (1, d), (2, c), (2, d), (3, c), (3, d)\}$$

$$f = \{(1, c); (2, d); (3, d)\}$$

But
$$\{(1, c), (1, d), (2, c)\}$$
Not a function (two values for 1, No value for 3

Ex $\mathbb{R} \times \mathbb{R}$

$$f = \{ (x, x^2) \; ; \; x \in \mathbb{R} \}$$

but $\{ (x^2, x) \; ; \; x \in \mathbb{R} \}$ ← Not a function!

$f : A \to B$

f maps A to B

if $(a, b) \in f \Rightarrow f(a) = b$

$E \subseteq A$

$f(E) := \{ f(x) : x \in E \}$ ← image of E.

$H \subseteq B$

$f^{-1}(H) := \{ x \in A : f(x) \in H \}$

inverse image of H.

(discussed last time)

$A = D(f)$ ← domain of f.

$R(f) = f(A) \subseteq B$ ← Range of f.

Def $f: A \to B$

f - injective (or one to one)

if $\forall x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

f - surjective (maps A onto B)

if $f(A) = B$, or $\forall y \in B$

$\exists x \in A : f(x) = y$.

f - bijective (f is bijection)

if f - surjective and injective

Ex $f(x) = x^2$ $f: \mathbb{R} \to \mathbb{R}$

Not injective, Not surjective

$f: \mathbb{R} \to \mathbb{R}^+ = \{x \in \mathbb{R}, x \geq 0\}$

surjective

$f: \mathbb{R}^+ \to \mathbb{R}^+$ bijection

Ex $A = \mathbb{R} \setminus \{1\}$

$f = \dfrac{2x}{x-1}$ ← injective, if

$$\frac{2x_1}{x_1-1} = \frac{2x_2}{x_2-1}$$

$$\Downarrow$$

$$x_1 = x_2$$

Find ~~Range~~ of f.

$$y = \frac{2x}{x-1}$$

$$yx - y = 2x$$

$$\cancel{x}(y-2) = \cancel{2x}\ y$$

$$x = \frac{y}{y-2}$$

Range $\mathbb{R} \setminus 2$

Inverse function

Def f - bijection of A onto B

$g := \{(b, a) \in B \times A : (a, b) \in f\}$

is function on B to A.

is inverse function of $\cancel{A}$

Notation f^{-1}

Note that $D(f) = R(f^{-1})$

$R(f) = D(f^{-1})$

and $b = f(a)$ iff $a = f^{-1}(b)$

if $f(x) = \frac{2x}{x-1}$ $\quad f: \mathbb{R} \setminus \{1\} \to \mathbb{R} \setminus \{2\}$

then $f^{-1}(y) = y/y-2$

Composition of functions

$f: A \to B$ and $g: B \to C$

if $R(f) \subseteq D(g) \subseteq B$

then composition of functions

$(g \circ f)(x) = g(f(x))$

$g \circ f : A \to C$

$f = 3x \qquad g = x^2 + 1$

$g \circ f \neq f \circ g$

$\parallel \qquad\qquad \parallel$

$9x^2 + 3 \qquad 3x^2 + 3$

$f = 1 - x^2 \qquad D(f) = \mathbb{R} \qquad R(g) = (-\infty,$

$g = \sqrt{x} \qquad D(g) = \mathbb{R}^+ \qquad R(g) = \mathbb{R}^+$

$\cancel{f \circ g} \quad g \circ f = \sqrt{1 - x^2}$

Defined only on $[-1, 1]$

$f \circ g = 1 - x$

but any way defined only on $\underline{\underline{x \geq 0}}$

$\mathbb{N} = \{1, 2, 3, \dots\}$

Defined by axioms

one of the main property is well ordering:

Every non empty subset has a least element.

i.e $S \subseteq \mathbb{N}$, $S \neq \emptyset$

$\exists k \in S : \forall m \in S \; k \leq m.$

Principle of Math. Induction.

$S \subseteq \mathbb{N}$:

1) $1 \in S$

2) If $k \in S \Rightarrow k+1 \in S$

$\Downarrow$

$S = \mathbb{N}$

Pf let $S \neq \mathbb{N}$ $\Rightarrow$

$\mathbb{N} \setminus S \neq \emptyset$

by well ordering there is

list ell $k \in \mathbb{N}/S$

$k > 1$

$\Rightarrow k-1$ - natural.

$k-1 \notin S$ $(k-1+1 = k \notin S)$

$k-1 \notin \mathbb{N}/S$ $(k-1 < k)$

contrad.