TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES
Home Work 1, due on Wednesday January 23
Instructor: Prof. Artem Zvavitch

Problem 1. *If tree fair dice are tossed, what is the probability that the sum is 6? what is the probability that one of the dice shows 1 given that the sum of all three is 6?*

Problem 2. *If tree fair dice are tossed, what is the average number of the sum is 6? What about 781 dice?*

Problem 3. *Suppose that you collect coupons! Suppose that each coupon obtained is, independent of what has been previously obtained, equally likely to be any of m types. Find the expected number of coupons one need to obtain in order to have at least one of each type.* **HINT:** *Let X be the number needed. It may help to present X by*

$$X = \sum_{i=1}^{m} X_i$$

where each X_i is geometric random variable (and represent how many coupons you need, AFTER you collected $i-1$ different coupons and trying to collect a coupon of a new type, i.e. to have i different coupons. For example $X_1 = 1$, X_2 is just a bit more tricky...).

Problem 4. *There four red cubes, six red balls and six yellow cubes in a box. How many yellow balls should you put in the box to make color and shape independent when the object from the box is selected at random?*

Problem 5. *Suppose X is a random variable such that*

$$\mathbb{P}(X=-2) = \mathbb{P}(X=-1) = \frac{1}{6},\ \mathbb{P}(X=0) = \frac{1}{2} \text{ and } \mathbb{P}(X=2) = \mathbb{P}(X=1) = \frac{1}{12}.$$

Please, find $\mathbb{E}[X]$, $\mathbb{E}[X^2]$ and $VarX = \mathbb{E}[X - \mathbb{E}X]^2$. Assume Y is an independent copy of X, please, find $\mathbb{E}[3X+6Y]$ and $\mathbb{E}[XY]$. Please, also compute $\mathbb{E}[X|X+Y = 0]$.

Problem 6. *Assume X is a uniform random variable on the interval $[-1,1]$(i.e. X has a density function $f(x) = \frac{1}{2}$, for $x \in [-1,1]$ and $f(x) = 0$ otherwise). Please, find cumulative distribution function $F(x) = \mathbb{P}(X \leq x)$, $\mathbb{E}[X]$, $\mathbb{E}[X^2]$ and $VarX$.*

Problem 7. *Prove that $\mathbb{E}X^2 \geq (\mathbb{E}X)^2$, also explain when/if equality is possible.*

Problem 8. *Assume X and Y are independent discrete random variables. Assume Y takes only positive numbers is it true that (if yes, present a proof, if not show a counterexample):*

- $\mathbb{E}\frac{X}{Y} = \frac{\mathbb{E}X}{\mathbb{E}Y}$?
- $\mathbb{E}\frac{X}{Y} = (\mathbb{E}X)(\mathbb{E}\frac{1}{Y})$?