TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES

Home Work 10 due on Wednesday April 24

Instructor: Prof. Artem Zvavitch

Problem 1. *Let* $\{B(t), t \geq 0\}$ *be a standard Brownian motion.*

- *What is the distribution of* $B(s) + B(t)$, $s \leq t$.
- *Compute the conditional distribution of* $B(s)$ *given that* $B(t_1) = A$ *and* $B(t_2) = B$, *where* $0 < t_1 < s < t_2$ *(i.e. find* $\mathbb{P}(B(s) \leq x | B(t_1) = A, B(t_2) = B)$*).*
- *Compute* $\mathbb{E}[B(t_1)B(t_2)B(t_3)]$, *for* $t_1 < t_2 < t_3$.
- *Compute* $\mathbb{P}\left(\max_{t_1 \leq s \leq t_2} B(s) > x\right)$.

Problem 2. *If* T_a *is a heating time of a standard Brownian motion process. Find* $\mathbb{P}(T_1 < T_{-1} < T_2)$.

Problem 3. *Let* $\{X(t), t \geq 0\}$ *be a Brownian motion with drift coefficient* μ *and variance* δ^2. *What is conditional distribution of* $X(t)$ *given that* $X(s) = c$ *when* $s < t$? *when* $s > t$?

Problem 4. *Suppose that* N *is a standard normal random variable and* $X = ae^{bN}$, *where* $a, b > 0$. *Find the density function of* X.