

TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES

Home Work 11 due on Wednesday May 1

Instructor: Prof. Artem Zvavitch

Problem 1. Suppose that N is a standard normal random variable and $X = ae^{bN}$, where $a, b > 0$. You have shown in the previous H.W. that the density f_X of X is

$$f_X(x) = \frac{1}{xb} \frac{1}{2\pi} e^{-\frac{(\log(x/a))^2}{2b^2}}.$$

Now, please, show that if $K > 0$, then

$$\int_0^\infty (x - K)^+ f_X(x) dx = ae^{b^2/2} \Phi\left(\frac{\log(a/K) + b^2}{b}\right) - K \Phi\left(\frac{\log(a/K)}{b}\right),$$

where Φ denotes the distribution function of N and $(x - K)^+$ is equal to zero if $x < K$ and $(x - K)$ if $x \geq K$.

Problem 2. A stock is presently selling at a price \$ 50 per share. After one time period, its selling price will (in present value dollars) be either \$ 150 or \$ 25. An option to purchase y units of the stock at time 1 can be purchased at cost cy .

- What should c be in order for there to be no sure win?
- Use the Arbitrage Theorem to find c so that there will be NO sure win (and verify above answers).
- Take $c = 4$ and show how you can guarantee a sure win.

Problem 3. The present price of a stock is 100. The price at time 1 will be either 50, 100 or 200. An option to purchase y shares of the stock at time 1 for the (present value) price ky cost cy .

- If $k = 120$ show that then an arbitrage opportunity occurs iff $c > 80/3$.
- If $k = 80$ show that there is not an arbitrage opportunity iff $c \in [20, 40]$.

Problem 4. The current price of stock is 100. Suppose that the logarithm of the price of the stock changes according to a Brownian motion with drift coefficient $\mu = 2$ and variance parameter $\delta^2 = 1$. Give the Black-Scholes cost of an option to buy the stock at time 10 for the cost of 100 per unit? 120 per unit? Assume that the continuously compounded interest is 5 percent.