TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES
Home Work 2, due on Wednesday February 06
Instructor: Prof. Artem Zvavitch

Problem 1. *The join density of random variables* X *and* Y *is given by*

$$f(x,y) = \frac{e^{-x/y}e^{-y}}{y}, \text{ where } 0 < x < \infty, 0 < y < \infty.$$

Compute $E[X|Y=y]$.

Problem 2. *Let* X_i, $i \geq 0$ *be independent and identically distributed random variable with probability math function* $p(k) = \mathbb{P}(X_i = k)$, *for* $k = 1, \cdots, m$ *(and* $\sum_{j=1}^{m} p(j) = 1$*).*
Find $\mathbb{E}[N]$, *where* $N = \min\{n : X_n = X_0\}$.

Problem 3. *Assume* X *and* Y *are a uniform random variables on the interval* $[-1, 1]$. *Find* $\mathbb{E}[X|X+Y=y]$.

Problem 4. *Assume* X *is a uniform random variable on the interval* $(0,1)$, *find* $\mathbb{E}[X^2|X < \frac{1}{2}]$.

Problem 5. *The join density of random variables* X *and* Y *is given by*

$$f(x,y) = \frac{e^{-y}}{y}, \text{ where } 0 < x < y, 0 < y < \infty.$$

Compute $\mathbb{E}[X^2|Y=y]$.

Problem 6. *If* $\mathbb{E}[X|Y=y] = constant$ *for all* y, *show that* $Cov(X,Y) = 0$.

Problem 7. *Prove that if* X, Y *are jointly continuous random variables, then*

$$\mathbb{E}X = \int_{-\infty}^{\infty} [\mathbb{E}X|Y=y] f_Y(y)dy.$$

Problem 8. *A coin having probability* p *of coming up heads is successively flipped until two of the most recent three flips are heads. Let* N *denote the number of flips. Find* $\mathbb{E}N$.

Problem 9. *A total of* 11 *people, including you, are invited for Artem's party. The times at which people arrive at the party are independent uniform* $(0,30)$ *(in minutes) random variables.*

- *Find the expected number of people who arrive before you. Also find the expected number of people who arrive after you.*
- *Find the variance of the number of people who arrive before you.*