TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES
Home Work 3, due on Wednesday February 13
Instructor: Prof. Artem Zvavitch

Problem 1. *Suppose that whether of not it rains today depends on previous weather conditions through the last three days. Show how this system may be analyzed by using Markov chain. How many states are needed?*

Problem 2. *Three white and three black balls are distributed in two urns in such a way that each contains three balls. We say that the system is in state* i , $i = 0, 1, 2, 3,$ *if the first urn contains* i *white balls. At each step, we draw one ball from each urn and place the ball drawn from the first urn into the second, and conversely with the ball from the second urn. Let* X_n *denote the state of the system after the nth step. Explain why* $\{X_n, n = 0, 1, 2, ...\}$ *is a Markov chain and calculate its transition probability matrix.*

Problem 3. *A Markov chain* $X_n, n = 0, 1, 2, \ldots$ *with states* $0, 1, 2,$ *has the transition probability matrix*

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

If $P(X_0 = 0) = P(X_0 = 1) = \frac{1}{4}$, *find* $\mathbb{E}X_3$.

Problem 4. *Suppose that coin* **1** *has probability* 0.7 *of coming up heads, and coin* **2** *has probability* 0.6 *of coming up heads. If the coin flipped today comes up heads, then we select coin* **1** *to flip tomorrow, and if it comes up tails, then we select coin* **2** *to flip tomorrow. If the coin initially flipped is equally likely to be coin* **1** *or* **2***, then what is the probability that the coin flipped on the third day after the initial flip is coin* **1**.

Problem 5. *Specify the classes of the following Markov chain and determine whether they are transient or recurrent:*

$$\mathbf{P} = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$