TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES
Home Work 4 due on Wednesday February 27
Instructor: Prof. Artem Zvavitch

Problem 1. *Consider a Marokov chain with state space* $\{1, 2, 3\}$ *and transition matrix*

$$\mathbf{P} = \begin{pmatrix} 0.4 & 0.2 & 0.4 \\ 0.6 & 0 & 0.4 \\ 0.2 & 0.5 & 0.3 \end{pmatrix}.$$

What is the probability in the long run that the chain is in state 1 *?*

Problem 2. *The Smiths receive the paper every morning and place it on a pile after reading it. Each afternoon, with probability* $1/3$*, someone takes all the papers in the pile and puts them in the recycling bin. Also, if ever there are at least five papers in the pile, Mr. Smith (with probability 1) takes the papers to the bin. Consider the number of papers in the pile in the evening.*

- *Model this by Markov Chain.*
- *Specify the classes of the following Markov chain and determine whether they are transient or recurrent.*
- *In the long run, what is the probability that there are no papers in the pile?*
- *After the long time, what would be the expected number of papers in the pile?*
- *Assume that the pile starts with* 0 *papers. What is the expected time until the pile will again have* 0 *papers?*

Problem 3. *Show that if state* i *is recurrent and state* i *does not communicate with state* j *then* $P_{ij} = 0$*. (This nice fact implies that once a process enters a recurrent class of states it can never leave that class)*

Problem 4. *Trials are performed in sequence. If the last two trials were successes then the next trial is success with probability* 0.8*; otherwise the next trial is a success with probability* 0.5*. In the long run, what proportion of trials are successes?*