

**TOPICS IN PROBABILITY THEORY AND
STOCHASTIC PROCESSES**

Home Work 5 due on Wednesday March 6

Instructor: Prof. Artem Zvavitch

Problem 1. Consider an organization of N employees (N is a HUGE number!). Each employee has one of three possible job classifications and changes classifications (independently) according to a Markov chain with transition probabilities

$$\mathbf{P} = \begin{pmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{pmatrix}.$$

What percentage of employees are in each classification?

Problem 2. For the Markov chain with states 1, 2, 3, 4 whose transitional probability

$$\mathbf{P} = \begin{pmatrix} .4 & .2 & .1 & .3 \\ .1 & .5 & .2 & .2 \\ .3 & .4 & .2 & .1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

find $f_{i,3}$ (probability that Markov chain EVER make transition from i to 3) and $s_{i,3}$ (expected number of times in state 3, given that we started from i), for $i = 1, 2, 3$.

Problem 3. Let $\phi_X(t) = \mathbb{E}e^{tX}$ be a moment generating function of a random variable X .

- Find $\left(\frac{d}{dt}\right)^n \phi_X(t)|_{t=0}$.
- Let X be an exponential distribution with parameter λ . Find $\phi_X(t)$ and use $\phi_X(t)$ to find $\mathbb{E}X$ and $\text{Var}(X)$.

Problem 4. In a branching process having $X_0 = 1$ and $\mu < 1$ show that the expected number of individuals ever exist in this population is given by $\mu/(1 - \mu)$. What if $X_0 = n$?

Problem 5. . Please find π_0 for the branching processes when

- $P_0 = 1/4, P_2 = 3/4$.
- $P_0 = 1/4, P_1 = 1/2, P_2 = 1/4$.
- $P_0 = 1/6, P_1 = 1/2, P_2 = 1/3$.