**TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES**
**Home Work 6 due on Wednesday March 13**
**Instructor: Prof. Artem Zvavitch**

**Problem 1.** *Suppose that you arrive at a single-teller bank to find five other customers in the bank, one being served and other four waiting in line. You join the end of the line. If the service times are all exponential with rate $\lambda$, what is the expected amount of time you will wait in the bank?*

**Problem 2.** *If $X_1, X_2, X_3$ are independent exponential random variables with rates $\lambda_1, \lambda_2$ and $\lambda_3$ find*

- $\mathbb{P}(X_1 < X_2 < X_3)$.
- $\mathbb{P}(X_1 < X_2 | \max(X_1, X_2, X_3) = X_3)$.
- $\mathbb{E}\min(X_1, X_2)$.
- $\mathbb{E}\max(X1, X2)$.
- $\operatorname{Var}\min(X1, X2)$.
- $E\ max(X1, X2, X3)$.

**Problem 3.** *If $X_1$ and $X_2$ are independent nonnegative random variables, show that*

$$P(X_1 < X_2)|\min(X_1, X_2) = t) = \frac{r_1(t)}{r_1(t) + r_2(t)}.$$

*where $r_i(t)$ is the failure rate function of $X_i$*

**Problem 4.** *Let $\{N(t), t \geq 0\}$ be a Poisson process with rate $\lambda$. Let $S_n$ denote the time of the nth event. Find*

- $\mathbb{E}S_4$,
- $\mathbb{E}[S_4|N(1) = 2]$,
- $\mathbb{E}[N(4) - N(2)|N(1) = 3]$.

**Problem 5.** *Consider two independent Poisson processes $\{N_1(t), t \geq 0\}$ with rate $\lambda_1$ and $\{N_2(t), t \geq 0\}$ with rate $\lambda_2$. Show that $\{N_1(t) + N_2(t), t \geq 0\}$ is a Poisson process with rate $\lambda_1 + \lambda_2$.*