TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES
Home Work 7 due on Wednesday March 20
Instructor: Prof. Artem Zvavitch

Problem 1. *Suppose that the number of calls per hour arriving at an answering service follows Poisson process with* $\lambda = 5$.

- *What is the probability that fewer than* 3 *calls come in the first hour?*
- *Suppose that seven calls arrive in the first hour. What is the probability that at least* 3 *calls will arrive in the second hour?*
- *The person answering the phones need to answer fifteen phone calls before going for a break. What is expected amount of time that the person will wait till her/his break?*
- *Suppose you know that exactly* 8 *calls arrived during the first two hours. What is the probability that exactly* 3 *arrived during the first hour?*
- *Suppose it is known that exactly* k *calls arrived in the first four hours. What is the probability that exactly* j *of them arrived in the first hour?*

Problem 2. *Let* $N_1(t)$ *and* $N_2(t)$ *(where* t *is a time in hours) be two independent Poisson processes with rates* λ_1 *and* λ_2*, respectively, counting the number of customers arriving in stores* 1 *and* 2*, respectively.*

- *What is the probability that a customer arrives in store* 1 *before any customer arrives in store* 2*?*
- *What is the probability that in the first hour, a total of exactly four customers have arrived at the two stores?*
- *Given that exactly four customers have arrived at the two stores, what is the probability that all four went in store* 1*?*
- *Let* T *denote the time of arrival of the first customer at store* 2*. Then* $N_1(T)$ *is the number of customers in store* 1 *at the time of the first customer arrival at store* 2*. Find the probability distribution of* $N_1(T)$ *(i.e. for each* k *find* $\mathbb{P}(N_1(T) = k)$*).*

Problem 3. *Cars pass a certain street location according to a Poisson process with rate* λ*. A woman who wants to cross the street at that location waits until she can see that no cars will come by in next* T *time units. Find the probability that her waiting time is 0. Find her expected waiting time (Hint: condition on the time of the first car).*

Problem 4. *Men and women enter a supermarket according to independent Poisson processes having respective rates two and four per minute. Starting at an arbitrary time, compute the probability that at least two men arrive before three women arrive*