TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES
Home Work 8 due on Monday April 8
Instructor: Prof. Artem Zvavitch

Problem 1. *Potential customer arrive at a single-server station in accordance with a Poisson process with rate λ. However, if the arrival finds n customers already in the station, the he/she will enter the system with probability $p_n \in [0,1]$ (i.e. with probability $1-p_n$ she/he will go away). Assuming an exponential service rate μ, set this up as a birth and death process and determine the birth and death rates.*

Problem 2. *Consider a birth and death process with birth rates $\lambda_i = (i+1)\lambda$, and death rates $\mu_i = i\mu$ for $i \geq 0$.*

(1) *Determine the expected time to go from state 0 to state 4.*
(2) *Determine the expected time to go from state 2 to state 5.*
(3) *Determine the variances in (1) and (2).*

Problem 3. *The birth and death process with parameters $\lambda_n = 0$ and $\mu_n = \mu$, $n > 0$ is called a pure death process. Find $P_{ij}(t)$.*

Problem 4. *Consider two machines, both of which have an exponential lifetime with mean $1/\lambda$. There is a single repairman that can service machines at exponential rate μ. Set up Kolmogorov backward equation.*

Problem 5. *A small barbershop, operated by a single barber, has a room for at most two customers. Potential customers arrive at a Poisson rate of three per hour, and the successive service times are independent exponential random variables with mean $1/4$ hour. What is the average number of customers in the shop? The proportion of potential customers that enter the shop? If the barber could work twice as fast, how much more business would he do?*